Page :

Date :

CH = 1

FORCES ACTING AT A POINT

MAGNITUDE AND DIRECTION OF The Resultant :

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\theta = \tan^{-1} \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

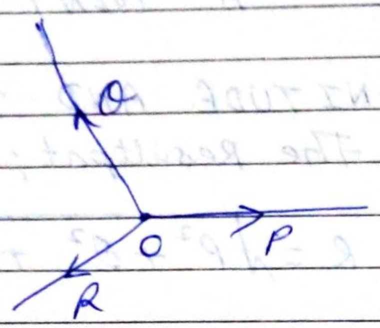
Components in any direction =

$$\frac{F \times \text{sine (other angle)}}{\text{Sine (sum of angles)}}.$$

Resolved Part of a force in any direction

= Force x Cosine of the angle which the force makes with that direction.

Triangle of forces:-



$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

λ-μ theorm

The resultant of two forces acting at a point O in direction OA and OB and represented in magnitude by $\lambda.OA$ and $\mu.OB$ is represented by $(\lambda+\mu).OC$ where C is a point in AB such that

$$\lambda.CA = \mu.CB$$

Lemi's theorm:-

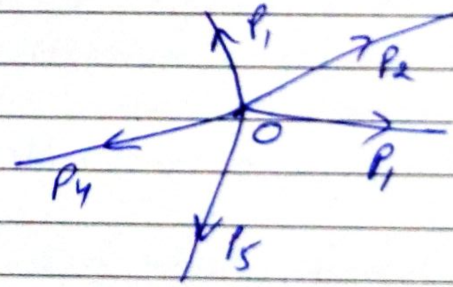
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Converse of LAmi's theorm:-

$$\frac{P}{BC} = \frac{Q}{CA} = \frac{R}{AB}$$

Polygon Law of forces.

If any no. of forces, acting at a point, be represented in magnitude and direction by sides of a polygon, taken in order, the force will be in equilibrium.



$$P_1 + P_2 + P_3 + P_4 + P_5 = 0$$

Equilibrium of Bodies Placed on a Smooth inclined Plane:-

A body of weight w is Placed on a smooth inclined Plane of inclination α and is supported by a force acting ~~hori~~ horizonally. To find the force and force and the reaction of the plane.

CH = 2

Parallel forces.

Resultant of two like Parallel forces Acting on a Rigid Body

:) Centre of the Parallel forces.

Theorem of Resolved Parts for Parallel forces.

Analogue of Lami's theorm:-

If three Parallel forces acting on a rigid body are in Equillibrium, each is Proportional to the distance b/w the other two.

Case-I when two forces are like.

Case-II when two forces are unlike.

Theorem of Resolved parts for Parallel forces.

The algebric sum of the resolved Parts of any two parallel forces in any direction is equal to the resolved part of their resultant in that direction.

Ch=3

MOMENTS

(I) moment of a force about a point:-

(I) motion of Translation.
(II) motion of Rotation.
(III) A combination of both.

This Product of $P \times p$ is called the moment of the force P about O.

Definition:- The moment of a force about a Point is the Product of the force and the Perpendicular distance of its line of action from the Point.

This moment of force P about O
$= P \times p$

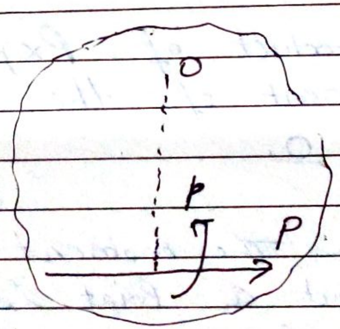
Cor-1

The moment of a force about a Point ~~x~~ in its own line of action is zero.

$$P \times p = 0$$

Cor-2 If the moment of a force about a point is zero, then either the force is zero, or it passes through the point.

Sign of moment of a force about a point:-



Geomentrical Representation of moment:-

The moment of force P about O

$= P \times ON$

$= AB \times ON$

$= 2 \times$ Area of ΔOAB

CHA-3

varignon's theorem:-

The algebric sum of the moments of two coplanar forces acting on a rigid body about any point in their plane is equal to the moment of their resultant about that point.

Centre of Number of Parallel forces:-

The centre of the system of parallel forces is the point where the resultant of all the forces of the given system acts, whatever the direction of the forces may be.

To find the centre of a system of Parallel forces Analytically.

(1) when the Points of application of the forces lie on a straight line.

(11) when the points of application of the forces lie in a Plane.

moment of a force About a line:-

moment of P about OX

$= P \sin\theta \times MN$

CH=4

COUPLES

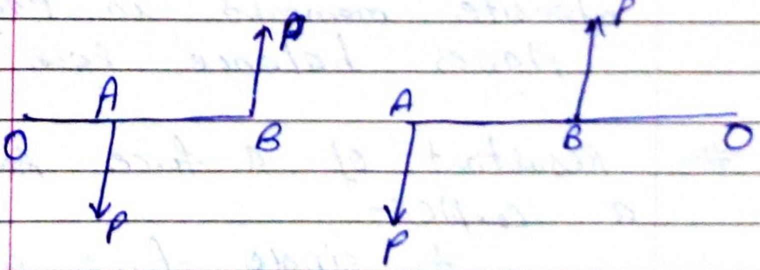
Two Equal and unlike parallel forces with different of action Are said to ~~form~~ a Couple.

moment of Couple:-

moment of Couple $(P.p)$

$=$ force $\times$ Arm of Couple.

⇒ The algebric Sum of the moments of two forces forming a Couple About any point in their Plane is Constant.



→ when O lies within AB

$= P \times AB = P.p$

Page :
Date :

\# Two Coplanar Couples of equal and opposite moments, balance each other.

(I) $\frac{P}{AB} = \frac{Q}{AD}$

(II) $P \times OB = Q \times CO$

\# Two couples in the same plane of equal moments and acting in the same are equivalent i.e are equal to each other.

\# Two Couples of equal and opposite moments in Parallel planes balance each other.

\# Resultant of a force And a couple:-

A single force and a Coplaner couple acting on a rigid body cannot produce

Page :
Date :

equilibrium but are equivalent to a single force equal and parallel to the given force.

⇒ Resolution of a force into a force And a couple:-

Any force is equivalent to an equal and parallel acting at an arbitrary point together with a couple of moment equal to the moment of the given force about that point.

Page :
Date :

CH=5

Analytical Conditions of Equilibrium of Co-planar forces.

→ If three forces acting on a rigid body keep it in equilibrium, they must be co-planar.

→ If three forces, acting in one plane upon a rigid body, keep it in equilibrium, they must either meet in a point or be parallel.

⇒ If the three coplanar forces are concurrent, then the following result is used which is the Lemi's theorem.

(I) They must be coplanar.

(II) They must be concurrent or parallel.

⇒ Trigonometrical theorem:- If G is a point in the side AB of a triangle OAB and if OG divides AB into two parts 'm' and 'n' and the angle AOB into two parts α and β then.

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$$

$$(m+n)\cot\theta = n\cot A - m\cot B$$

where θ is the angle OGB.

First form:-

The necessary and sufficient conditions that a system of Coplanar forces acting on a rigid body be in equilibrium are.

(I) The algebric sum of the resolved parts of the forces along the direction OX is zero.

(II) The algebraic sum of resolved parts of the forces along a direction OY is zero.

(III) The algebric sum of the moments of the forces about any point in their plane is zero.

⇒ A system of Co-planar forces shall be in equilibrium if the algebric sum of the moments of all the forces about any three non-collinear points in their plane vanish separatly.

⇒ A system of Coplanar forces shall be in equilibrium if the algebric sum of the moments of all the forces about any two points in their plane vanish seprately and the algebric sum of their resolved parts be zero in any direction not perpendicular to the line joining the two points.

Rules:-

(I) A neat and clear figure should be drawn, carefully marking all the forces acting on the body.

(II) Write down any geometrical relations that are provided in the figure.

(III) Take two suitable perpendicular directions as axes ox and oy. Generally the horizontal and vertical directions are found quite convenient. In the case of problems where inclined planes are sometimes found more suitable.

(IV) Equate to zero the sum of the resolved parts of the force along ox as well as along oy.

(V) Equate to zero the algebraic sum of the moments of the forces about a suitable point.

(VI) If a rod is placed on a peg, the reaction will be perpendicular to the floor or the wall.

(VIII) If a rod is placed inside the sphere, the reaction at the ends pass through the centre of the sphere.

(ix) Reaction at the hinge is of adjusting type. If the body is in equilibrium under three coplanar forces and the two forces are intersecting, then the reaction at the hinge also passes through the point of intersection.

CH-6

FRICTION

The Property of roughness of bodies by virtue of which a force is exerted to resist the motion of one upon another is called friction and the force exerted is called the force of friction.

→ Statical friction:— The force of friction called into play when there is equilibrium is called statical friction.

→ Dynamical friction:— The force of friction called into play when there is motion is called Dynamical friction.

Laws of Statical friction:—

(I) Statical friction acts in the direction opposite to that in which the body tends to move.

(ii) The magnitude of this force is just sufficient to prevent the body from moving.

⇒ Laws of limiting friction:—

(I) The direction of the limiting friction is opposite to that in which the body is about to move.

(II) The magnitude of limiting friction of the point of contact between two bodies bears a constant ratio to normal reaction at that point.

(III) The limiting friction is independent of the shape and area of the surfaces in ~~Cont~~ contact.

FRICTION

The Property of roughness of bodies by virtue of which a force is exerted to resist the motion of one upon another is called friction and the force exerted is called the force of friction.

→ Statical friction:— The force of friction called into play when there is equilibrium is called statical friction.

→ Dynamical friction:— The force of friction called into play when there is motion is called Dynamical friction.

Laws of Statical friction:—

(I) Statical friction acts in the direction opposite to that in which the body tends to move.

(ii) The magnitude of this force is just sufficient to prevent the body from moving.

⇒ Laws of limiting friction:—

(I) The direction of the limiting friction is opposite to that in which the body is about to move.

(II) The magnitude of limiting friction of the point of contact between two bodies bears a constant ratio to normal reaction at that point.

(III) The limiting friction is independent of the shape and area of the surfaces in ~~Cont~~ contact.

Co-efficient of Friction:–

The Constant ratio which the limiting friction bears to the normal reaction is called the Co-efficient of friction.

It usually denoted by the letter μ.

$$\boxed{\frac{F}{R} = \mu}$$

$$\boxed{F = \mu R}$$

Resultant Reaction:

$$S = \sqrt{R^2+F^2}$$

$$= \sqrt{R^2+\mu^2 R^2}$$

$$= R\sqrt{1+\mu^2}$$

Angle of Friction:–

$$\boxed{\tan\lambda = \frac{F}{R} = \mu}$$

CONE OF FRICTION:–

The Cone of friction is the Cone which has the point of Contact as its vertex, the normal as its axis and λ as its Semi-vertical angle.

To find the force required to support a heavy body on a rough inclined plane, inclined to the horizontal at an angle greater than the angle of friction, the force acting in a vertical plane through a line of the greatest slope through the body.

Cor-1 $$P' = w\,\frac{\sin(\alpha-\lambda)}{\cos\lambda}$$

Cor-2 $$P' = w\,\frac{\sin(\alpha-\lambda)}{\cos\lambda}$$

CH=7

CENTRE OF GRAVITY

Centre of Gravity of a No. of Particles lying in a straight line:-

$$\bar{x} = \frac{\sum_{j=1}^{n} m_j x_j}{\sum_{j=1}^{n} m_j}$$

CENTRE OF GRAVITY OF NO OF PARTICLES LYING IN A PLANE:-

$$\bar{x} = \frac{\Sigma m_j x_j}{\Sigma m_j}$$

$$\bar{y} = \frac{\Sigma m_j y_j}{\Sigma m_j}$$

CENTRE OF GRAVITY OF A UNIFORM ROD :-

~~Here~~ C.G of the whole Rod AB is at G. i.e middle point.

Centre of gravity of a thin Uniform lemina in the form of a Parallelogram:–

The Centre of Gravity of a thin uniform lamina in the form of a Parallelogram is the point of intersection of its diagonals.

Centre of gravity of a thin uniform TRIANGULAR Lamina:–

C.G lies at the point G ie the point of intersection of the medians AD and BE which is the centroid of the triangle.

Centre of Gravity of a Lamina in the form of a Trapezium:–

The C.G of the trapezium is the same as the C.G of these weights at E and F

To find the centre of gravity of a Body By integration?–

$$\bar{x} = \frac{\int x\,dm}{\int dm}$$

$$\bar{y} = \frac{\int y\,dm}{\int dm}$$

Centre of Gravity of a thin uniform Rod:–

Let $\bar{x}$ be the distance of C.G from A

$$\bar{x} = \frac{a}{2}$$

(1) $$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

(2) $$ds = \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta$$

Page :
Date :

(4) $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Page :
Date :

CH=8

VIRTUAL WORK

=) A force acting on a body is said to do work, when its point of application is displaced from one position to the other along the line of action of force.

(I) The tension of an inextensible string (Pull)

(II) The thrust of an inextensible rod (Pull).

(III) The mutual action and reaction b/w two bodies whose equilibrium is being considered together.

(IV) The reaction R of any smooth surface with which the body is in contact.

(V) Reaction at the points of constant of a fixed surface.

on which a body rolls without sliding.

(VI) The mutual reaction b/w two bodies which roll upon one another.

CH=9

FORCES IN THREE DIMENSIONS.

If three forces acting at a point O are represented in magnitude and direction by the edges OA, OB and OC of a parallelopiped, then their resultant also acts at O and is represented in magnitude and direction by the diagonal of the parallelopiped.

$$R = \sqrt{X^2+Y^2+Z^2}$$

Composition of Couple:

moment $G = \sqrt{L^2+M^2+N^2}$

Direction cosines

$$\left\langle \frac{L}{G}, \frac{M}{G}, \frac{N}{G} \right\rangle$$

Page :
Date :

(1) ⇒ Central Axis:

If a system of forces is reduced to a single force $\vec{R}$ and a single couple such that axis of the couple is coincident with the line of action of force $\vec{R}$, then that line is called Central axis.

(2) Wrench:

Suppose a system of forces is reduced to a single force $\vec{R}$ and a couple of moment $\vec{k}$ whose axis coincides with the direction of the acting force, then $\vec{R}$ and $\vec{k}$ taken together are called wrench.

Page :
Date :

Working Rule to find central axis:

(I) Write the eqn of the line along which the force f_r acts in form

$$\frac{x-x_r}{l_r} = \frac{y-y_r}{m_r} = \frac{z-z_r}{n_r}$$

(II) The values of L_r, M_r, N_r are obtained by equating the cofficients of $\hat{i}, \hat{j}, \hat{k}$ in the eqn.

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_r & y_r & z_r \\ X_r & Y_r & Z_r \end{vmatrix}$$

(III) The eqn of the central axis is:-

$$\frac{L-(yZ-zY)}{X} = \frac{M-(zX-xZ)}{Y} = \frac{N-(xY-yX)}{Z}$$

Pitch of the wrench

$$p = \frac{K}{R} = \frac{LX + MY + NZ}{X^2 + Y^2 + Z^2}$$

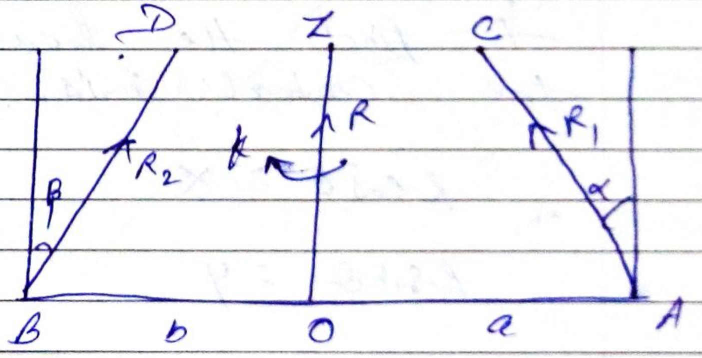
The system reduces to a single force if

$$LX + MY + NZ = 0$$

CH = 10

WRENCHES

⇒ To show that a given system of forces can be replaced by two forces, equivalent to the given system, in an infinite number of ways and that the tetrahedron formed by the two forces is of constant volume.



$$R_1 \cos\alpha + R_2 \cos\beta = R$$

$$R_1 \sin\alpha - R_2 \sin\beta = 0.$$

To find the resultant wrench of two given forces R_1 and R_2 inclined at an angle θ.

The axes of two given wrenches intersect at right angles. Their intensities are X and Y and their pitches are p_x and p_y; If the pitches are given, to find the locus of the central axis.

$$R\cos\theta = X$$

$$R\sin\theta = Y$$

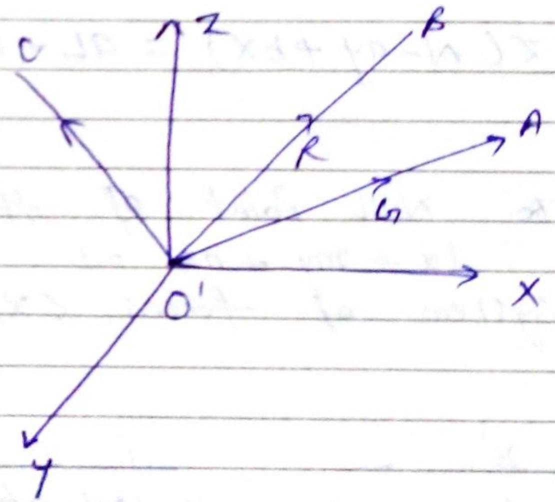
$$\tan\theta = Y/X$$

CH=11

NULL LINES AND NULL PLANES.

NULL LINES :-

Null lines of a given system of forces are those lines about which the moment of the system vanishes.



NULL PLANE :-

The Plane in which all the null lines lie is called the null plane of the point O'.

The Point O' itself is called null point.

⇒ Equation to the null Plane of a given Point (a, b, c) referred to any axes OX, OY, OZ :-

$$x(L - bZ + cY) + y(M - cX + aZ) + z(N - aY + bX) = aL + bM + cN$$

To Null Point of the Plane $lx + my + nz = 1$ for the system of forces (X, Y, Z, L, M, N)

$$\frac{x'}{X - (nM - mN)} = \frac{y'}{Y - (lN - nL)} = \frac{z'}{Z - (mL - lM)} = \frac{1}{lX + mY + nZ}$$

the Points (x', y', z') which is null Point.

CH = 12

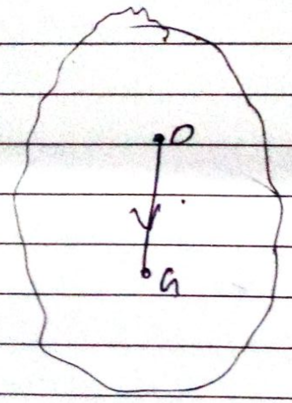
STABLE, UNSTABLE AND NEUTRAL EQUILIBRIUM

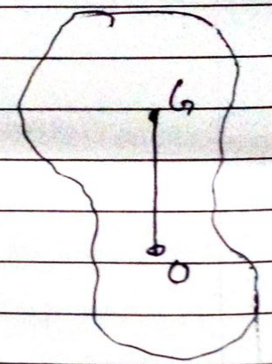
Equilibrium of Bodies:—

(I) Weight of the body acting vertically downwards through its centre of gravity.

(II) Tension in the String.

Position of Equilibrium:—





Conditions of Stability of Equilibrium:—

(I) Let the height z of the C.G of the system above a fixed point or fixed horizontal plane be expressed as a function of some variable θ say, $z = f(\theta)$

(II) Solve the equation $\frac{dz}{d\theta} = 0$

the values of θ so obtained will given the position of equilibrium as under:

(a) If $\frac{d^2z}{d\theta^2}$ is positive for any value of θ, then the equilibrium is stable.

(b) If $\frac{d^2z}{d\theta^2}$ is negative, then the equilibrium is

Page :

Date :

unstable for that value of θ.

(i) If $\frac{d^2 z}{d\theta^2}$ is Positive, then the Equilibrium is unstable.

(ii) If $\frac{d^2 z}{d\theta^2}$ is negative, then the Equilibrium is stable.