Maa Omwati Degree Collage Hassanpur

NOTES

Subject:- Optics II

B.SC Sem:-IV

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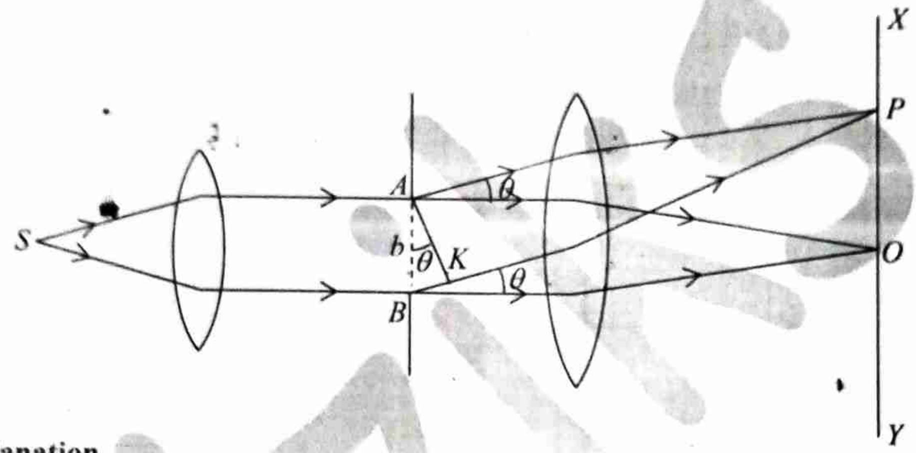
3.1 Fraunhofer's diffraction at a single slit

Let a parallel beam of monochromatic light of wavelength λ be incident normally upon a narrow slit of width $AB = b$ placed perpendicular to the plane of the paper. Let the diffracted light be focused by a convex lens L on a screen XY placed in the focal plane of the lens. The diffraction pattern obtained on the screen consists of a central bright band, having alternative dark and weak bright bands of decreasing intensity on both sides.



Explanation

In terms of wave theory, a plane wave front is incident normally on the slit AB. According to the Huygen's principle, each point on AB sends out secondary wavelets in all directions. The rays proceeding in the same direction as the incident rays are focused at O, while those diffracted through an angle θ are focused at P. Let us find out the resultant intensity at P.

Let AK be the perpendicular to BK. As the optical path from the plane AK to P are equal, the path difference between the wavelets from A to B in the direction

$$\text{Path difference} = BK = AB\sin\theta = b\sin\theta$$

The corresponding phase difference $= \dfrac{2\pi}{\lambda}(b\sin\theta)$

Let the width AB of the slit is divided into n equal parts. The amplitude of vibration at P. Due to the waves from each part will be the same, say equal to a. The phase difference between the waves from any two consecutive parts is $\frac{1}{n}\left(\frac{2\pi}{\lambda}b\sin\theta\right)=\delta$ (say).

Hence the resultant amplitude at P is $R=\dfrac{a\sin\left(\frac{n\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}=\dfrac{a\sin\left(\frac{\pi b\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi b\sin\theta}{n\lambda}\right)}$

Let $\quad \dfrac{\pi b\sin\theta}{\lambda}=\alpha \quad \Rightarrow R=\dfrac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}=\dfrac{a\sin\alpha}{\frac{\alpha}{n}} \qquad \because \frac{\alpha}{n}$ is small

$$R=\frac{n.a.\sin\alpha}{\alpha}$$

As $n\to\infty$, $\alpha\to 0$ but the product na remains finite

Thus Resultant Amplitude at P due is $R=A\dfrac{\sin\alpha}{\alpha}$ let $na=A$

Thus resultant intensity at P; $\quad I=R^2=A^2\left(\dfrac{\sin\alpha}{\alpha}\right)^2$

The constant of proportionality being taken as unity for simplicity

Condition for Maximum and Minimum intensity

Direction for Minimum Intensity

For minimum intensity $I=0 \Rightarrow \dfrac{\sin\alpha}{\alpha}=0 \Rightarrow \sin\alpha=0$ but $\alpha\neq 0$.

Thus $\alpha=\pm m\pi$ where; m has an integral value 1, 2, 3 except zero

$$\frac{\pi b\sin\theta}{\lambda}=\pm m\pi$$

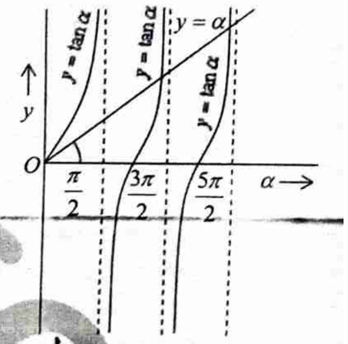
$\Rightarrow b\sin\theta=\pm m\lambda \qquad$ where $m=1,2,3$

Direction for Maximum Intensity

For direction of maximum intensity $\dfrac{dI}{d\alpha}=0$

$$\Rightarrow \frac{d}{d\alpha}\left[A^2\frac{\sin^2\alpha}{\alpha^2}\right]=0\Rightarrow A^2\left(\frac{2\sin\alpha}{\alpha}\right)\frac{\alpha\cos\alpha-\sin\alpha}{\alpha^2}=0$$

$$\Rightarrow \frac{\alpha\cos\alpha-\sin\alpha}{\alpha^2}=0\Rightarrow 2\cos\alpha-\sin\alpha=0\Rightarrow \quad \alpha=\tan\alpha$$



The equation is solved graphically by plotting the curve

$y=\alpha$ and $y=\tan\alpha$

the points of intersection gives (approximately)

$$\alpha=0,\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}=0,1.430\pi,2.462\pi,3.471\pi.....$$

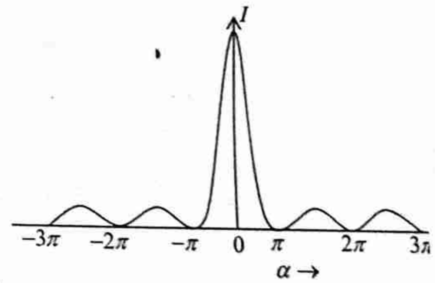
Substituting the value of α into the expression of I

The intensity of the central maxima $I_0=A^2\left(\dfrac{\sin 0}{0}\right)=A^2$

The intensity of the first maxima $I_1=A^2\left[\dfrac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right]^2=\dfrac{A^2}{22}$

The intensity of the second maxima

$$I_2=A^2\left[\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right]^2=\frac{A^2}{61}$$



Thus the ratio of intensities of the successive maxima

$$I_0,I_1,I_2....=1:\frac{1}{22}:\frac{1}{61}:\frac{1}{121}$$

Clearly most of the incident light is concentrated in

the central [illegible] which occurs in the direction $\alpha=0$ or $\dfrac{\pi b\sin\theta}{\lambda}$ [illegible]

the same direction as the incident light

Effect of Slit Width on Diffraction Pattern

If slit is made narrower

Since first minimum on either side of the central maximum occurs in the direction θ, given by equation $b\sin\theta = \pm\lambda$

When the slit is narrowed (b is reduced), the angle θ increases which means the central maximum becomes wider. When the slit-width is as small as wavelength $(b = \lambda)$, the first minimum occurs at $\theta = 90^0$, which means central maxima fills the whole space i.e. condition of uniform illumination.

Example: A monochromatic light with a wavelength of $\lambda = 600\,nm$ passes through a single slit which has a width of $0.800\,mm$.

(a) What is the distance between the slit and the screen be located if the first minimum in the diffraction pattern is at a distance $1.00\,mm$ from the center of the screen?

(b) Calculate the width of the central maximum.

Solution: (a) The general condition for destructive interference is

$$\sin\theta = m\frac{\lambda}{b} \qquad m = \pm1, \pm2, \pm3, ...$$

For small θ, we employ the approximation $\sin\theta \approx \tan\theta = \frac{y}{D}$, which yields $\frac{y}{D} \approx m\frac{\lambda}{b}$

The first minimum corresponds to $m = 1$. If $y_1 = 1.00\,mm$, then

$$L = \frac{by_1}{m\lambda} = \frac{(8.00\times10^{-4}\,m)(1.00\times10^{-3}\,m)}{1(600\times10^{-9}\,m)} = 1.33\,m$$

(b) The width of the central maximum is $W = 2y_1 = 2(1.00\times10^{-3}\,m) = 2.00\,mm$

3. Diffraction of Light

The spreading out of a wave when it passes through a narrow opening is usually referred to as diffraction, and the intensity distribution on the screen is known as the diffraction pattern.

The diffraction phenomena are usually divided into two categories: ***Fresnel diffraction*** and ***Fraunhofer diffraction***.

In the Fresnel class of diffraction the source of light and the screen are, in general, at a finite distance from the diffracting aperture In the Fraunhofer class of diffraction, the source and the screen are at infinite distances from the aperture; this is easily achieved by placing the source on the focal plane of a convex lens and placing the screen on the focal plane of another convex lens. The two lenses effectively moved the source and the screen to infinity because the first lens makes the light beam parallel and the second lens effectively makes the screen receive a parallel beam of light.

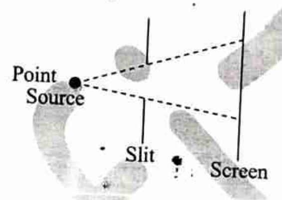


Fig: Fresnel diffraction

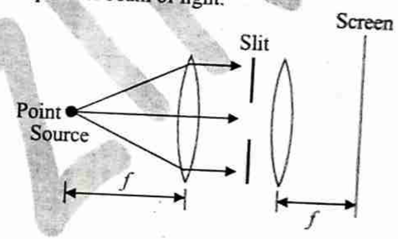


Fig: Fraunhofer diffraction

Ans. 38: 1.93

Solution: The path difference is $\Delta = d\sin\theta$

for $D >> y$, $\sin\theta \cong \tan\theta = \frac{y}{D}$

$$\therefore \quad \Delta = \frac{yd}{D}$$

where, $y = 1.8\times10^{-2}\ m$, $d = 1.5\times10^{-4}\ m$ and $D = 1.4\ m$

$$\therefore \quad \Delta = \frac{\left(1.8\times10^{-2}\ m\right)\left(1.5\times10^{-4}\ m\right)}{1.4\ m} = 1.93\times10^{-6}\ m \Rightarrow \Delta = 1.93\ \mu m$$

Example: A monochromatic light is incident on a single slit of width $0.800\,mm$ and a diffraction pattern is formed at a screen which is $0.800\ m$ away from the slit. The second order bright fringe is at a distance $1.60\,mm$ from the centre of the central maximum. What is the wavelength of the incident light?

Solution: The general condition for destructive interference is

$$\sin\theta = m\frac{\lambda}{b} \approx \frac{y}{D}$$ where small-angle approximation has been made.

Thus the position of the m^{th} order dark fringe measured from the central axis is

$$y_m = m\frac{\lambda D}{b}$$

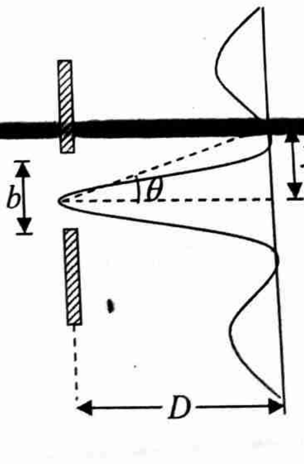
Let the second bright fringe be located halfway between the second and the third dark fringe. That is $y_{2b} = \frac{1}{2}(y_2 + y_3) = \frac{1}{2}(2+3)\frac{\lambda D}{b} = \frac{5\lambda D}{2b}$

The approximate wavelength of the incident light is then

$$\lambda = \frac{2by_{2b}}{5D} = \frac{2(0.800\times10^{-3}m)(1.60\times10^{-3}m)}{500(0.800m)} = 6.40\times10^{-7}m$$

Example: Light of wavelength $580\ nm$ is incident on a slit having a width of $0.30\ mm$. The viewing screen is $2\ m$ from the slit. Find the positions of the first dark fringes and the width of the central bright fringe. What if the slit width is increased by an order of magnitude of $3.0\ mm$? What happens to the diffraction pattern?

Solution: To analyze the problem, note that the two dark fringes that flank the central bright fringe corresponds to $n = \pm 1$



$$\therefore \quad b\sin\theta_{dark} = \pm\frac{\lambda}{b}$$

$$\Rightarrow \quad \sin\theta_{dark} = \pm\frac{5.80\times10^{-7}m}{0.3\times10^{-3}m} = \pm1.933\times10^{-3}$$

since, $y_1 = D\tan\theta_{dark}$

For small angle $\theta_{dark} \cong \tan\theta_{dark} \cong \sin\theta_{dark} \cong \frac{y_1}{D}$

$\therefore y_1 = D\sin\theta_{dark} = (2m)\left(\pm 1.933\times 10^{-3}\right) = \pm 3.87\times 10^{-3} m$

The width of the central bright fringe is $W = 2|y_1| = 2\times 3.87\times 10^{-3} m = 7.74\ mm$

Note that this value is much greater than the width of the slit.

We expect that angles at which the dark bands appear will decrease as b increases

Thus the diffraction pattern narrows.

For $b = 3.0\ mm$, the sine of the angle θ'_{dark} for the $n = \pm 1$ dark fringes are

$$\sin\theta_{dark} = \pm\frac{\lambda}{b} = \pm\frac{5.8\times 10^{-7}\ m}{3\times 10^{-3}\ m} = \pm 1.933\times 10^{-4}$$

$\therefore \quad y_1 \cong D\sin\theta_{dark} = (2.0\ m)\left(\pm 1.933\times 10^{-4}\right) = \pm 3.87\times 10^{-4}\ m$

The width of the central bright fringe is

$$W = 2y_1 = 2\times 3.87\times 10^{-4}\ m$$

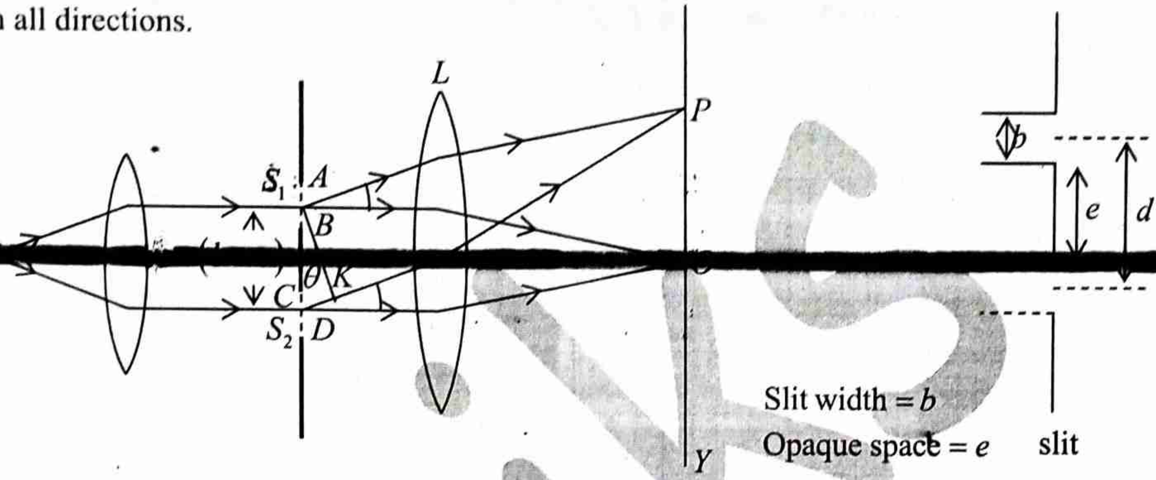
$\Rightarrow \quad W = 0.774\ mm$

Notice that this is smaller than the width of the slit

3.2 Fraunhofer's Diffraction at a Double Slit

In Double Slit diffraction experiment, the pattern obtained on the screen is the diffraction pattern due to a single slit on which a system of interference fringes is superposed.

By Huygen's principle every point in the slits AB and CD sends out secondary wavelets in all directions.



From the theory of diffraction at a single slit, the resultant amplitude due to wavelets diffracted from each slit in a direction θ is $R = A\dfrac{\sin\alpha}{\alpha}$, where $\alpha = \dfrac{\pi b\sin\theta}{\lambda}$.

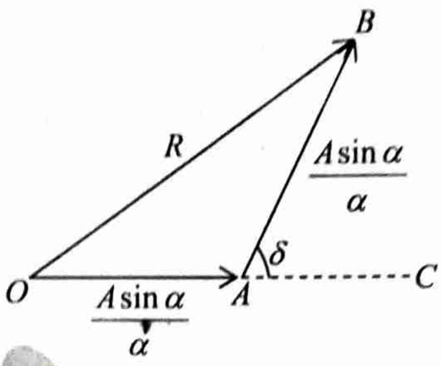
Consider the two slits as equivalent to two coherent sources placed at the middle points S_1 and S_2 of the slits and each sending wavelets of amplitude $A\dfrac{\sin\alpha}{\alpha}$ in a direction θ.

Consequently, the resultant amplitude at a point P on the screen will be the result of interference between two waves of same amplitude $A\dfrac{\sin\alpha}{\alpha}$, and having a phase difference δ (say) and path difference $S_2K = (b+e)\sin\theta$.

Phase difference $\delta = \dfrac{2\pi}{\lambda}(b+e)\sin\theta$

The resultant amplitude R

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta$$



$$R^2=\left(A\frac{\sin\alpha}{\alpha}\right)^2(2+2\cos\delta)\Rightarrow R^2=\frac{A^2\sin^2\alpha}{\alpha^2}4\cos^2\frac{\delta}{2}$$

Let $\beta=\frac{\delta}{2}\Rightarrow\beta=\frac{\pi(b+e)\sin\theta}{\lambda}$

Therefore the resultant intensity at P is

$$I=R^2=4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Thus the intensity in the resultant pattern depends on two factors

(i) $\frac{\sin^2\alpha}{\alpha^2}$, which gives diffraction pattern due to each individual slit and

(ii) $\cos^2\beta$, which gives interference pattern due to diffracted light waves from the two slits.

The diffraction term $\frac{\sin^2\alpha}{\alpha^2}$ gives central maximum in the direction $\theta=0$, having alternate minima and subsidiary maxima of decreasing intensity on either side the minima are obtained in the direction given by

$$\sin\alpha=0 \quad\Rightarrow\quad \alpha=\pm m\pi$$

$$\frac{\pi b\sin\theta}{\lambda}=\pm m\pi \quad \text{where } m=1,2,3,.... \text{ (but not zero)}$$

Thus the condition of diffraction minimum is

$$b\sin\theta=\pm m\lambda \qquad (m=1,2,3,...)$$

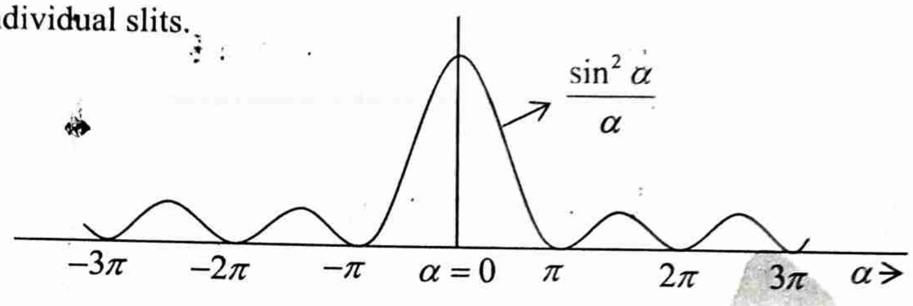
The interference term $\cos^2\beta$ gives a set of equidistant dark and a bright fringe as in Young's double slit interference experiment. The bright fringe are obtained in the direction

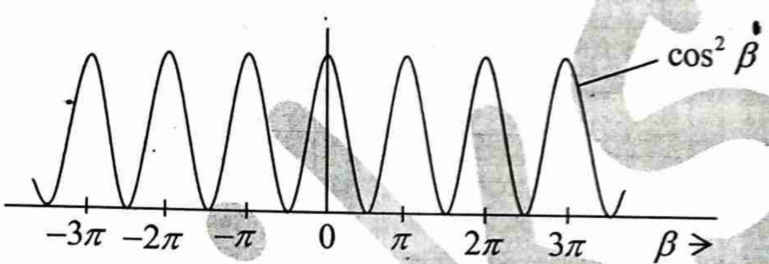
$$\cos^2\beta=1 \quad\Rightarrow\beta=\pm n\pi \quad\Rightarrow\quad \frac{\pi}{\lambda}(b+e)\sin\theta=\pm n\pi$$

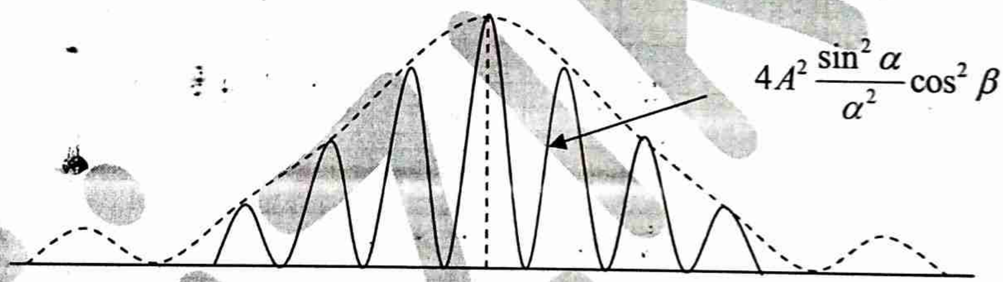
Thus the condition of maximum is

$$(b+e)\sin\theta=\pm n\lambda \qquad \text{where } n=0,1,2,....$$

The entire pattern may be considered as consisting of interference fringes due to light from both slits, the intensities of these fringes being governed by the diffraction occurring at the individual slits.







Effect of Slit Width on Diffraction Pattern

(1) Effect of increasing the slit width

If we increase the slit width b, the envelope of the fringe pattern changes so that it central peak is sharper. The fringe spacing, which depends on slit separation, does not change. Hence less interference maxima now fall within the central diffraction maximum.

(2) Effect of increasing the distance between slits

If b is kept constant and the separation d between them is increased, the fringes becomes closer together, the envelope of the pattern remaining unchanged. Thus more interference maxima fall within the central envelope.

Absent Orders

For certain values of d certain interference maxima become absent from the pattern. Suppose for some value of θ the following conditions are simultaneously satisfied.

$$(b+e)\sin\theta = \pm n\lambda \qquad \text{Interference maxima}$$

$$b\sin\theta = \pm m\lambda \qquad \text{Diffraction minima}$$

Thus according to the first condition there should be an interference maximum in the direction θ, but according to the second condition there is no diffracted light in this direction. Therefore, the interference maximum will be absent in this direction.

$$\frac{b+e}{b} = \frac{n}{m}$$

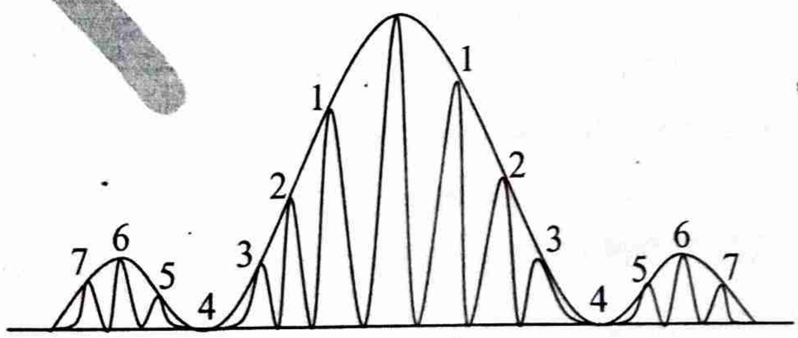
(i) If $e = b$, then $n = 2m = 2,4,6,....$ Since $m = 1,2,3,....$

Thus 2^{nd}, 4^{th} and 6^{th} order interference maxima will be absent, i.e. they will coincide with 1^{st}, 2^{nd}, 3^{rd} order diffraction minima. Thus the central diffraction maximum will have three interference (zero order and two first orders) maxima.

(ii) If $e = 2b$, then $n = 3m = 3,6,9,....$

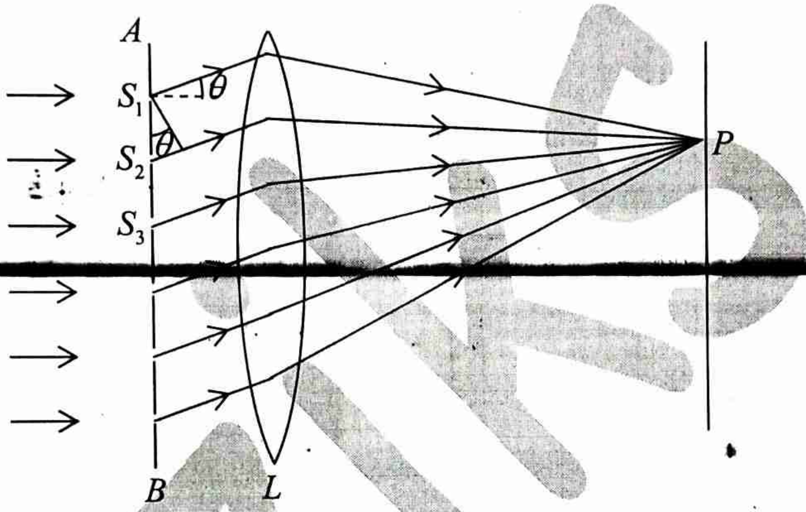
Thus the 3^{rd}, 6^{th}, 9^{th} order interference maxima will coincide with 1^{st}, 2^{nd}, 3^{rd} order diffraction minima. Thus the central diffraction maximum will have five interference maxima.

(iii) If $e = 3b$, then $n = 4m = 4,8,12,....$ that is 4^{th}, 8^{th}, 12^{th} .... order interference maxima will coincide with 1^{st}, 2^{nd} and 3^{rd} order diffraction minima. Thus the central diffraction maximum will have seven inference maxima as shown below.



3.3 Diffraction Grating

A diffraction grating is an arrangement equivalent to a large number of parallel slits of equal widths and separated from one-another by equal opaque spaces. It is made by ruling a large number of fine, equidistant and parallel lines on an optically plane glass plate with a diamond point. The rulings scatter the light and are effectively opaque while the un-ruled parts transmit light and act as slits.



Let AB be the section of a plane transmission grating, the lengths of the slits being perpendicular to the plane of the paper. Let b be the width of each slit and e the width of each opaque space between the slits, then $d = b + e$ is called the grating element.

Let a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. By the theory of Fraunhofer diffraction at a single slit, the wavelets from all points in a slit diffracted in a direction θ are equivalent to a single wave of amplitude $A\dfrac{\sin\alpha}{\alpha}$, starting from the middle point of the slit where $\alpha = \dfrac{\pi b \sin\theta}{\lambda}$.

Thus if N be total number of slits in the grating, the diffracted rays from all the slits are equivalent to N parallel rays, one each from the middle points $S_1, S_2, S_3 \ldots$ of the slits.

Path difference between the rays from the slits S_1 and S_2 is

$$S_2K = S_1S_2 \sin\theta = (b+e)\sin\theta$$

The corresponding phase difference $=\frac{2\pi}{\lambda}(b+e)\sin\theta = 2\beta$

Hence the resultant amplitude in the direction θ is $R = A\frac{\sin\alpha}{\alpha}.\frac{\sin N\beta}{\sin\beta}$

The resultant intensity is $I = R^2 = A^2\left(\frac{\sin^2\alpha}{\alpha^2}\right)\left(\frac{\sin^2 N\beta}{\sin^2\beta}\right)$

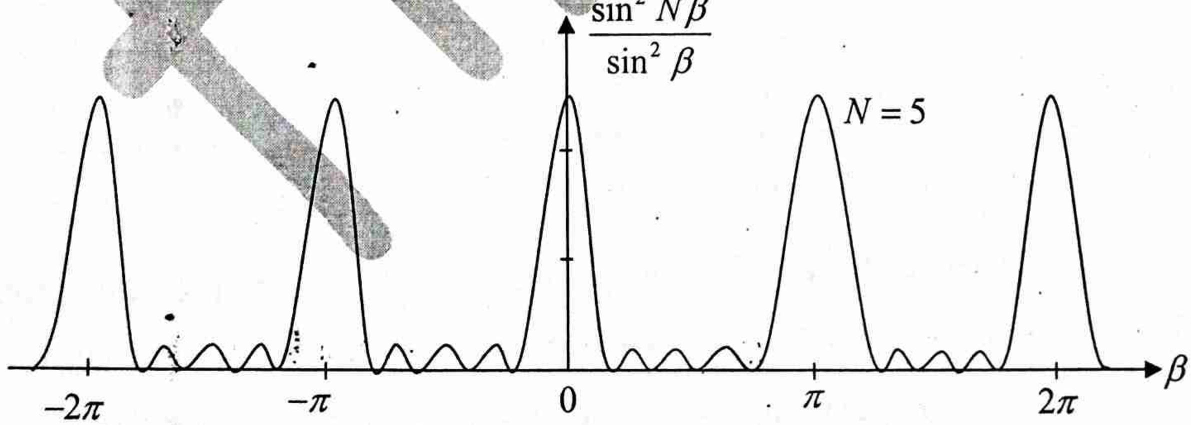
The first term represents the diffraction pattern produced by a single slit whereas the second term represents the Interference pattern produced by N equally spaced slits. For $N=1$ the above equation reduces to a single slit diffraction pattern and for $N=2$, to the double-slit diffraction pattern.

Following figure shows the plot of $\frac{\sin^2 N\beta}{\sin^2\beta}$ as a function of β for $N=5$. As the value of N becomes very large, the above function becomes narrow and very sharply peaked at $\beta = 0, \pi, 2\pi....$

Between the two peaks, the function vanishes when

$$\beta = \frac{m'\pi}{N} \qquad m' = +1, +2, +3, ... \text{ but } m' \neq 0, +N, +2N.....$$

which are referred to as interference minima.



Position of Maxima and Minima

Condition for Principle Maxima

When the value of N is very large, one obtains intense maxima when $\sin\beta = 0$. This condition arises when

$$\beta = \pm n\pi \qquad \text{where } n = 0,1,2,3.....$$

so $\sin N\beta = 0$

Thus $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$ (indeterminate form), thus

$$\lim_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi}\frac{N\cos N\beta}{\cos\beta} = \pm N$$

Thus intensity

$$I = \frac{A^2\sin^2\alpha}{\alpha^2}N^2 = N^2 I_0\frac{\sin^2\alpha}{\alpha^2}$$

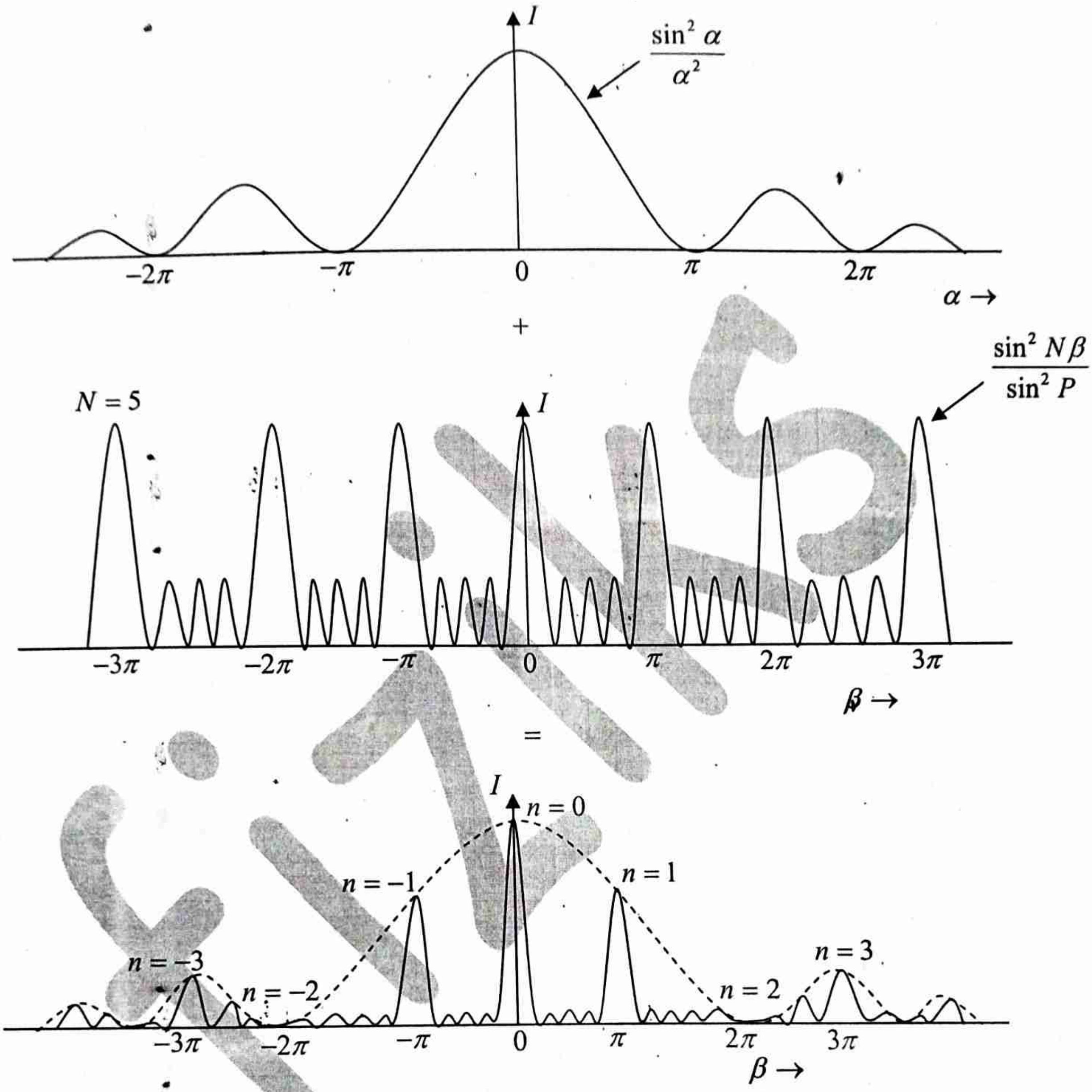
Such maxima are most intense and are known as principal maxima. Physically, at these maxima the fields produced by each of the slits are in phase, and therefore, they add and the resultant field is N times the field produced by each of the slits.

They are obtained in the direction given by $\beta = \pm n\pi$

$$\frac{\pi}{\lambda}(b+e)\sin\theta = \pm n\pi$$

$$(b+e)\sin\theta = \pm n\lambda \qquad \text{where } n = 0,1,2...$$

This is the condition of maxima where for $n = 0$, we get the zero order principle maximum. For $n = \pm 1, +2 + 3.....$ we obtain the first, second, third- order principal maxima respectively. The $\pm$ sign show that there are two principal maxima for each order lying on either side of the zero order maximum.



Condition for Minima

The intensity is zero when $\sin N\beta = 0$ but $\sin\beta \neq 0$ then $\dfrac{\sin N\beta}{\sin\beta} = 0$

($\sin\beta = 0$ gives maximum intensity). Thus minimum are obtained in the directions given by

$$\sin N\beta = 0 \;\Rightarrow\; N\beta = \pm m'\pi \qquad \Rightarrow N\frac{\pi}{\lambda}(b+e)\sin\theta = \pm m'\pi$$

Thus the condition of minimum intensity is

$$N(b+e)\sin\theta = \pm m'\lambda$$

where m' takes all integral values except $0, N, 2N,... nN$. Because these value of m' makes $\sin\beta = 0$, which gives principal maxima.

It is clear from above that $m' = 0$ gives the principal maximum and $m' = 1,2,3.. (N-1)$ give minima and then $m' = N$ give again a principal maximum. Thus there are $(N-1)$ minima between two consecutive principal maxima.

Condition for Secondary Maxima

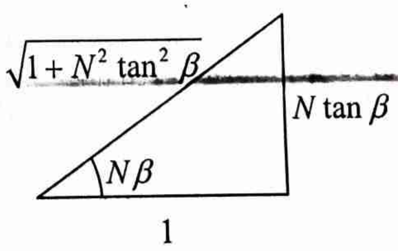
As there are $(N-1)$ minima between two consecutive principal maxima, there must be $(N-2)$ other maxima between two principal maxima these are called secondary maxima. Their positions are obtained by differentiating the formula of intensity I with respect to β and equating it equal to zero. Thus

$$\frac{dI}{d\beta} = \frac{A^2\sin^2\alpha}{\alpha^2}2\left[\frac{\sin N\beta}{\sin\beta}\right]\frac{N\cos N\beta\sin\beta - \sin N\beta\cos\beta}{\sin^2\beta} = 0$$

$$N\cos n\beta\sin\beta = \sin N\beta\cos\beta \qquad \Rightarrow \tan N\beta = N\tan\beta \qquad \text{----(1)}$$

To find the value of $\frac{\sin^2 N\beta}{\sin^2\beta}$ under the condition (1), we make use of the triangle shown in figure (1), this gives

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2\tan^2\beta}{(1+N^2\tan^2\beta)} = \frac{N^2}{(1+N^2\tan^2\beta)\cos^2\beta}$$



$$= \frac{N^2}{\cos^2\beta + N^2\sin^2\beta} = \frac{N^2}{1+(N^2-1)\sin^2\beta}$$

This gives $\qquad \sin N\beta = \frac{N\tan\beta}{\sqrt{1+N^2\tan^2\beta}}$

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This shows that the intensity of the secondary maxima is proportional to $\frac{N^2}{1+\left(N^2-1\right)\sin^2\beta}$, where the intensity of the principal maxima is proportional to N^2 so,

$$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}}=\frac{1}{1+\left(N^2-1\right)\sin^2\beta}$$

Hence greater the value of N, the weaker are secondary maxima. In an actual grating, N is very large. Hence these secondary maxima are not visible in the grating spectrum.

Condition for absent spectra

A particular principal maximum may be absent if it corresponds to the angle which also determines the minimum of the single-slit diffraction pattern.

Principal maxima in the grating spectrum are obtained in the direction given by

$$(b+e)\sin\theta=n\lambda \qquad \text{-----(1)}$$

where n is the order of maximum.

The minima in a single slits pattern are obtained in the direction given by

$$b\sin\theta=m\lambda \qquad m=1,2,3... \qquad \text{-----(2)}$$

If both (1) and (2) are satisfied simultaneously, a particular maximum of order n will be missing in the grating spectrum.

$$\frac{b+e}{b}=\frac{n}{m}$$

this is the condition for the spectrum of the order n to be absent.

If $e=b$ then $n=2m=2,4,6...$ i.e. 2nd, 4th, 6th, order spectra will be absent

If $e=2b$ then $n=3m=3,6,9...$ i.e. 3rd, 6th, 9th...... order spectra will be absent

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Example: Monochromatic light beam a helium-neon laser $(\lambda = 632.8\ nm)$ is incident normally on a diffraction grating containing 6000 grooves per centimeter. Find the angles at which the 1st and 2nd order maxima are observed. What if we look for the 3rd order maxima? Do we find it?

Solution: First we must calculate the slit separation, which is equal to the inverse of the number of grooves per centimeter.

$$d = \frac{1}{6000}\ cm = 1.667\times10^{-4}\ cm = 1667\ nm$$

For the 1st order maximum $(n=1)$, we obtain

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{632.8\ nm}{1667\ nm} = 0.3796 \Rightarrow \theta_1 = 22.31^\circ$$

For the 2nd order maximum $(n=2)$, we find

$$\sin\theta_2 = \frac{2\lambda}{d} = \frac{2(632.8\ nm)}{1667\ nm} = 0.7592 \Rightarrow \theta_2 = 49.39^\circ$$

For 3rd order maximum $(n=3)$, we find

$$\sin\theta_3 = \frac{3\lambda}{d} = \frac{3(632.8\ nm)}{1667\ nm} = 1.139$$

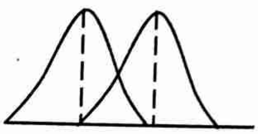
because $\sin\theta$ cannot exceed unity, this does not represent a realistic solution. Hence only zeroth 1st & 2nd order maxima are observed for this situation

3.4 Rayleigh Criterion of Resolution and Resolving Power

The resolving power of an optical instrument represents its ability to produce distinctly separate spectral lines of light having two or more close wavelengths.

3.4.1 Rayleigh's Criterion of Resolution

Lord Rayleigh proposed the following criterion for resolution which has been universally adopted. "Two spectral lines of equal intensities are just resolved by an optical instrument when the principal maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.



3.4.2 Resolving Power of a Grating

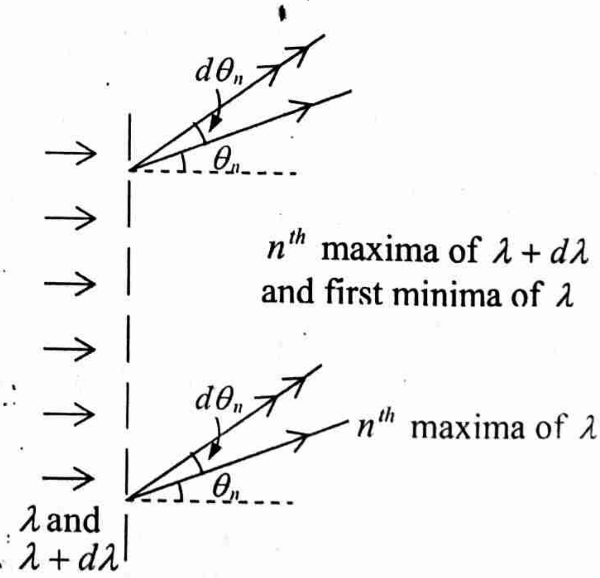
The resolving power of a grating represents its ability to form separate spectral lines for wavelengths very close together. It is defined by $R = \frac{\lambda}{\Delta\lambda}$ where $\Delta\lambda$ is the separation of the two wavelengths which the grating can just resolve; the smaller the value of $\Delta\lambda$, larger the resolving power.

Let a parallel beam of light of two wavelengths of λ and $\lambda + d\lambda$ be incident normally on the grating. If the n^{th} principal maximum of λ is formed in the direction θ_n, we have

$$(b+e)\sin\theta_n = n\lambda \qquad (1)$$

where $(b+e)$ is the grating element.

Let the first minimum adjacent to the n^{th} maximum be obtained in the direction $(\theta_n + d\theta_n)$.



The equation for the minima is

$$N(b+e)\sin\theta = m'\lambda \qquad (2)$$

here $\theta = \theta_n + d\theta_n$

where N is the total number of rulings on the grating and m' takes all integral values except $0, N, 2N, \ldots, nN$ because these values of m' give $0^{th}, 1^{st}, 2^{nd}, \ldots, n^{th}$ principal maximum respectively.

Clearly, the first minimum adjacent to the principal maximum in the direction of θ increasing will be obtained for $m' = (nN+1)$. Therefore, if this minimum is obtained in the direction $\theta_n + d\theta_n$, we have from equation (2)

$$N(b+e)\sin(\theta_n + d\theta_n) = (n.N+1)$$

$$(b+e)\sin(\theta_n + d\theta_n) = \frac{n.N+1}{N}\lambda \qquad (3)$$

By Rayleigh's criterion, the wavelengths λ and $(\lambda + d\lambda)$ are just resolved by the grating when the n^{th} maximum of $(\lambda + d\lambda)$ is also obtained in the direction $\theta_n + d\theta_n$. Then, we have from equation (1)

$$(b+e)\sin(\theta_n + d\theta_n) = n(\lambda + d\lambda) \qquad (4)$$

Comparing equation (3) and (4), we get

$$\frac{n.N+1}{N}\lambda = n(\lambda + d\lambda)$$

$$n.N\lambda + \lambda = Nn\lambda + +Nnd\lambda$$

$$\lambda = Nnd\lambda$$

$$\frac{\lambda}{d\lambda} = nN$$

But $\frac{\lambda}{d\lambda}$ is the resolving power R of the grating.

Therefore, $R = \frac{\lambda}{d\lambda} = nN$

The above expression may be written as

$$R = nN = \frac{N(b+e)\sin\theta_n}{\lambda}$$

But $N(b+e)$ is the total width of the grating. Hence at a particular angle of diffraction θ_n, the Resolving Power is directly proportional to the total width of the ruled space on the grating.

3.4.3 Difference between Dispersive Power and Resolving Power

The dispersive power of a diffraction grating gives us an idea of the angular separation between the lines of a spectrum produced by a grating, it is measured by $\frac{d\theta}{d\lambda}$ where $d\theta$ is the angular separation between two spectral lines whose wavelengths differ by $d\lambda$. The value of Dispersive Power is given by

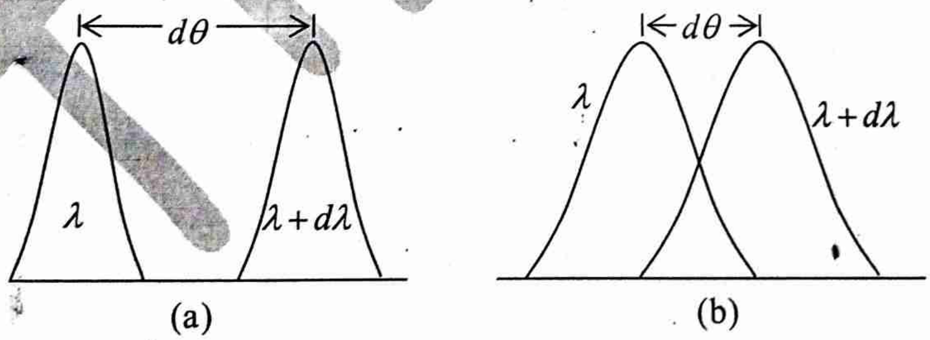
$$\frac{d\theta}{d\lambda}=\frac{n}{(b+e)\cos\theta}$$

Thus, higher is the order n of the spectrum or closer are the rulings on the grating (i.e. smaller the value of $(b+e)$ greater is the dispersive power.

The resolving power of the grating, on the other hand, expresses the degree of closeness which the spectral lines can have and yet be distinguished as two. It is measured by $\frac{\lambda}{d\lambda}$, where $d\lambda$ is the smallest wavelength λ. The value of Resolving Power is given by

$$\frac{\lambda}{d\lambda}=nN$$

where, N is the total number of rulings on the grating and n is order of diffraction. Thus greater is the width of the ruled surface, higher is the resolving power. The higher resolving power results in sharp maxima.



Resolving power is greater in case (a) than case (b).

Example: When a gaseous element is raised to a very high temperature, the atoms emit radiation having discrete wavelengths. Tow strong components in the atomic spectrum have wavelengths of $5890\,\overset{0}{A}$ & $5896\,\overset{0}{A}$.

(a) What resolving power must a grating have if these wavelengths are to be distinguished?

(b) To resolve these lines in the second-order spectrum, how many slits of the grating must be illuminated?

Solution:

(a) Resolving power is defined as

$$R = \frac{\lambda}{\Delta\lambda} \text{ where } \lambda = \frac{\lambda_1 + \lambda_2}{2} = \frac{5890 + 5896}{2} = 5893\,\overset{0}{A}$$

$$\Delta\lambda = \lambda_2 - \lambda_1 = 5896 - 5890 = 6\,\overset{0}{A}$$

$$\therefore R = \frac{5893\,\overset{0}{A}}{6\,\overset{0}{A}} = 982 \quad \Rightarrow R = 982$$

(b) Resolving power also defined as $R = nN$ where n is the order of spectrum and N is number of slits

$$N = \frac{R}{n} = \frac{982}{2} = 491 \text{ slits}$$

Example: A beam of light is incident normally on a diffraction grating of width 1 *cm*. It is found that at 30°, the n^{th} order diffraction maximum for $\lambda_1 = 600$ Å is super-imposed on the $(n+1)^{th}$ order for $\lambda_2 = 500$ Å. How many lines per cm does the grating have? Find out whether the first order spectrum from such a grating can be used to resolve the wavelengths $\lambda_3 = 5800$ Å and $\lambda_4 = 5802$ Å?

Solution:

Condition for diffraction maxima is

$$(b+e)\sin\theta = n\lambda$$

For given two wavelength the maxima condition can be written as

$$(b+e)\sin 30^0 = n\lambda_1 \quad and \quad (b+e)\sin 30^0 = (n+1)\lambda_2$$

Taking ration of the two, we get

$$n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{5000}{1000} = 5$$

The grating element is

$$(b+e)\sin 30^0 = 5\times 500\overset{0}{\text{A}} \Rightarrow b+e = 5\times 10^{-5}\,cm$$

Number of grating is

$$N = \frac{Grating \quad width}{Grating \quad element} = \frac{W}{b+e} = \frac{1}{5\times 10^{-5}} = 20000$$

Resolving power of this Grating is

$$R = \frac{\lambda}{d\lambda} = nN.$$

For 1st order spectrum, the resolving power of the given grating is

$$R = nN = 20000.$$

Whereas to resolve the wavelengths $\lambda_3 = 5800$ Å and $\lambda_4 = 5802$ Å the required resolving power is

$$R = \frac{\lambda}{d\lambda} = \frac{5800}{2} = 2900.$$

Resolving power of the given grating (20000) is more than required (2900). Thus, given grating will resolve the wavelengths $\lambda_3 = 5800\overset{0}{A}$ and $\lambda_4 = 5802\overset{0}{A}$.

MCQ (Multiple Choice Questions)

Q1. Consider Fraunhofer diffraction pattern obtained with a single slit illuminated at normal incidence. At the angular position of the first diffraction minimum, the phase difference (in radian) between the wavelets from the opposite edges of the slit is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) 2π (d) π.

Q2. A plane transmission grating has 800 lines and $0.5\ cm$ width. Light consisting of two doublets I and II falls on it normally. The mean wavelengths of doublets I and II are $6000\,\mathring{A}$ and $4200\,\mathring{A}$, respectively, and the wavelength separation for both is $6\,\mathring{A}$. Which one of the following is correct in first-order spectrum ?

(a) Both the doublets are not resolved

(b) Doublet I is not resolved, but doublet II is resolved

(c) Both the doublets are resolved

(d) Doublets I is resolved, but doublet II is not resolved

Q3. In diffraction grating, slit width is b and ruling separation is e. If second order spectrum is to be avoided, one should choose:

(a) $b=\frac{e}{2}$ (b) $b=2e$ (c) $b=\sqrt{2e}$ (d) $b=e$

Q4. A diffraction grating has N lines and grating element is $(b+e)$. For wavelength λ, what is the maximum resolving power possible?

(a) $\frac{N(b+e)}{\lambda}$ (b) $\frac{(b+e)}{(N\lambda)}$

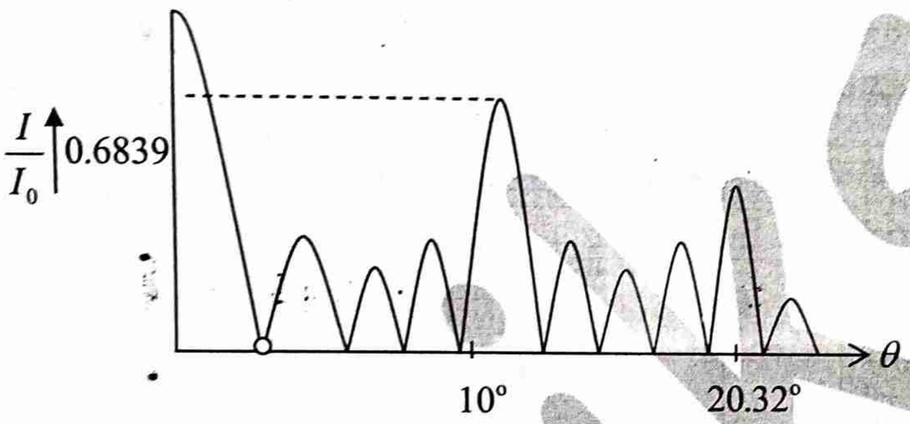
(c) $(b+e)\frac{N^2}{\lambda}$ (d) $(b+e-\lambda)^2\frac{N}{\lambda^2}$

Q5. A $10 mW$ laser beam $(\lambda_0 = 0.6\,\mu m)$ is launched on a lens of focal length $10\ cm$. The diameter of the laser beam is $4\ mm$. The area of the focused spot is approximately given by:

(a) $2.25\times10^{-10}\ cm^2$ (b) $2.25\times10^{-6}\ cm^2$

(c) $2.25\times10^{-8}\ cm^2$ (d) $2.25\times10^{-9}\ cm^2$

Q6.



A typical N slit diffraction grating intensity distribution is shown in the figure above. Assume $\lambda = 6\times10^{-5}\ cm$. The value of N given by:

(a) 4 (b) 5 (c) 6 (d) 7

Q7. A screen is placed at a distance D from a narrow diffracting slit of width $b << D$. What is the separation between central diffraction maxima and the first minima, if wavelength of diffraction light λ?

(a) $\frac{Db}{\lambda}$ (b) $\frac{\lambda^2}{D}$ (c) $\frac{D\lambda}{b}$ (d) $\frac{b\lambda}{D}$

Q8. The angular separation between two wavelengths λ and $\lambda + d\lambda$ in a diffraction grating is directly proportional to:

(a) frequency of light (b) grating element

(c) width of grating (d) wavelength of light

Q9. In a plane diffraction grating with N elements, the intensity of the diffracted beam is:

(a) Proportional to $\sqrt{N}$ (b) Proportional to N

(c) Proportional to N^2 (d) independent of N

Q10. Which one of the following plane transmission gratings of width W, and number of lines per centimeter N, will have the maximum resolving power in the first order?

(a) $W = 1\ cm,\ N = 5000$ (b) $W = 1.5\ cm,\ N = 4000$

(c) $W = 2\ cm,\ N = 2400$ (d) $W = 3\ cm,\ N = 1500$

Q11. A grating which would be most suitable for constructing a spectrometer for the visible and ultraviolet region should have:

(a) 100 lines/cm (b) 1000 lines/cm

(c) 10,000 lines/cm (d) 10,00,000 lines/cm

Q12. A parallel beam of light of wavelength $600\ nm$ gets diffracted by a single-slit of width $0.2\ mm$. The angular divergence of the first maxima of diffracted light is

(a) $6\times10^{-3}\ rad$ (b) $3\times10^{-3}\ rad$

(c) $4.5\times10^{-2}\ rad$ (d) $9.0\times10^{-2}\ rad$

Q13. A single slit is used to observe diffraction pattern with red light. On replacing the red light with violet light the diffraction pattern would:

(a) remain unchanged (b) become narrower

(c) become broader (d) disappear

Q14. The minimum number of lines in a grating which will just resolve the spectral lines of wavelengths $5880\ \overset{0}{A}$ and $5886\ \overset{0}{A}$ in the second order is:

(a) 491 (b) 981 (c) 2940 d) 2943

Q15. In the far-field diffraction pattern of a single slit under polychromatic illumination, the first minima with the wavelength λ_1 is found to be coincident with the third minima at λ_2 than the relationship between these two wavelengths is:

(a) $3\lambda_1 = 0.3\lambda_2$ (b) $3\lambda_1 = \lambda_2$

(c) $\lambda_1 = 3\lambda_2$ d) $0.3\lambda_1 = 3\lambda_2$

MSQ (Multiple Select Questions)

Q16. Fraunhofer diffraction pattern of multiple identical parallel slits is observed. If the number of slits is decreased, keeping the total aperture width and the slit size constant, how are the fringes affected?

(a) The fringes become brighter
(b) The fringes become less bright
(c) The fringes become broader
(d) The fringes become sharper

Q17. Which of the following can be diffracted?

(a) Radio waves
(b) Sound waves
(c) Microwave
(d) X -ray

Q18. Yellow light of intensity I_0 is used in a single slit diffraction experiment with slit width of 0.6 mm if yellow light is replaced by Red light of same intensity I_0 , then the observed pattern will reveal,

(a) that the central maximum is broader
(b) Intensity of central maxima reduces
(c) Intensity of central maxima remain same
(d) First minima appears at higher angle

NAT (Numerical Answer Type)

Q19. A slit of width d is placed in front of a lens of focal length $0.5\ m$ and is illuminated normally with light of wavelength $5.89\times10^{-7}\ m$. The first diffraction minima on either side of the central diffraction maximum are separated by $2\times10^{-3}\ m$. The width d of the slit is ________ $\times10^{-4} m$.

Q20. The wave lengths of sodium D lines are $589.59\ um$ and $588.99\ um$ The minimum number of lines that a grating must have in order to resolve these lines in the first order spectrum is ____________.

Q21. In a diffraction grating experiment, the grating has 10^{-5} rulings. Then, in the $\lambda = 5000\ \overset{0}{A}$ region of the spectrum and in the second order, the grating can resolve two line with a wavelength difference of ___________ $\overset{0}{A}$

Q22. The angular dispersion of two spectral lines from a plane diffraction grating in first order spectrum is 3° the dispersion in the second order spectrum for the same spectral lines will be approximately ____________ degree.

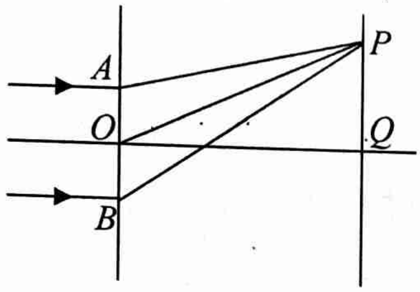
Q23. In a single-sit Fraunhofer diffraction set-up used with light of wavelength $4000\ \overset{0}{A}$, the distance D between the central maximum and first minimum is found to be $0.3\ cm$. In the same setup, if the wavelength of light used is changed to $6000\ \overset{0}{A}$, the corresponding value of D will be ____________ cm..

Solution-MCQ (Multiple Choice Questions)

Ans. 1: (c)

Solution: The figure depicts Fraunhofer diffraction pattern.

For P to be first minimum, the phase difference between OA or OB is π.



$\therefore$ Phase difference from opposite edges $= 2\pi$ radian

Ans. 2: (b)

Solution: Resolving power of grating is given as $\dfrac{\lambda}{d\lambda} = nN$

where $n =$ order of spectrum, $N =$ number of lines in grating

For first order $n = 1$, $\dfrac{\lambda}{d\lambda} = N$

(i) when $\lambda = 6000\,\overset{0}{A}$, $d\lambda = 6\,\overset{0}{A}$ then $N = \dfrac{\lambda}{d\lambda} = \dfrac{6000}{6} = 1000$

Thus, the number of lines required lines on grating is 1000 but actual was 800, so it cannot be resolved.

(ii) when $\lambda = 4200\,\overset{0}{A}$, $d\lambda = 6\,\overset{0}{A}$ $\Rightarrow N = \dfrac{4200}{6} = 700$

Thus, number of required lines on grating is 700 but actual line is 800. It is resolved.

Ans. 3: (d)

Solution: If slit width is e and ruling separation is d, then the direction of interference maxima are given as $(b+e)\sin\theta = n\lambda$ ..... (i)

The direction minima are given as $b\sin\theta = m\lambda$ ..... (ii)

In equation (i) and (ii) n and m are integers. If the value of b and e are such that the both equations are satisfied simultaneously for the same value of θ, then the position of certain interference maxima correspond to the diffraction minima at the same position on the screen.

For second order missing $\frac{n}{m} = 2 \Rightarrow \frac{b+e}{b} = 2 \Rightarrow b+e = 2b \Rightarrow b = e$

Ans. 4: (a)

Solution: The direction of interference maxima for a grating is given as

$$(b+e)\sin\theta = n\lambda \Rightarrow \qquad n = \frac{(b+e)\sin\theta}{\lambda}$$

Resolving power of grating $R = nN$

n is order of diffraction , N is total number of ruling

From equation (i) it is clear that maximum resolution of diffraction grating occurs at $\theta = 90^0, \sin\theta = 1$

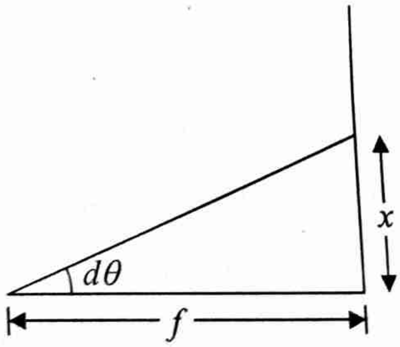
Maximum resolution $= \frac{(b+e)}{\lambda}N$

Ans. 5: (d)

Solution: If D is diameter of the laser beam, then $D\sin\theta = \lambda$

If θ is very small, then $\sin\theta = d\theta$

$$Dd\theta = \lambda \Rightarrow \frac{xD}{f} = \lambda \Rightarrow x = \frac{f\lambda}{D}$$

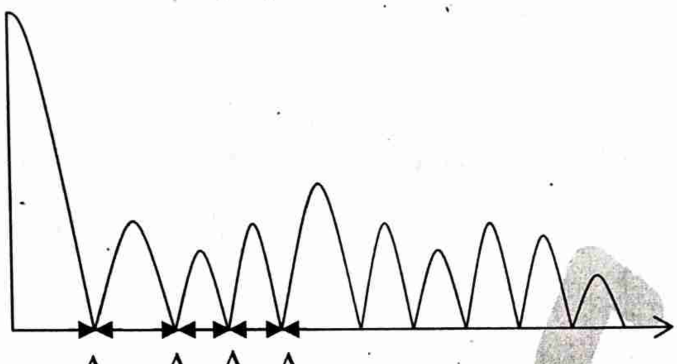


So, area

$$\pi x^2 = \pi\left(\frac{f\lambda}{D}\right)^2 = 3.14\times\left(\frac{0.6\times10^{-6}\times10^{-11}}{4\times10^{-3}}\right)^2 \text{cm}^2 = 3\times10^{-19}\ \text{cm}^2$$

Ans. 6: (b)

Solution: Between two principal maxima of grating have N slit there is $(N-1)$ minima.



From the figure there are four minima represented by A_1, A_2, A_3, A_4

Hence, $N-1=4 \quad \Rightarrow N=5$

Ans. 7: (c)

Solution: If b is width of narrow slit then minima of diffraction pattern is given as

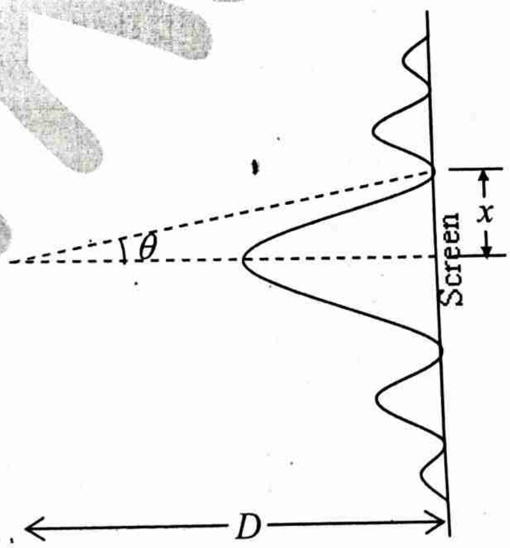
$$b\sin\theta = m\lambda$$

For first minima $m=1$, So, $b\sin\theta=\lambda$

If θ is very small $\sin\theta \cong \tan\theta \cong \dfrac{x}{D}$

Thus, $\dfrac{bx}{D}=\lambda \Rightarrow x=\left(\dfrac{\lambda D}{b}\right)$

$\Rightarrow$ Separation between first minima and central maxima is $\dfrac{\lambda D}{b}$



Ans. 8: (d)

Solution: The diffraction pattern for a light of wavelength λ and stiff width e separated by a distance b is given as

$$(b+e)\sin\theta_n = n\lambda \Rightarrow (b+e)\cos\theta_n . d\theta_n = nd\lambda$$

$$\Rightarrow \quad d\theta_n = \frac{nd\lambda}{(b+e)\cos\theta_n}$$

Ans. 9: (c)

Solution: In plane grating the intensity is proportion to square of number of lines drawn on the grating $I \propto N^2$

Ans. 10: (b)

Solution: The resolving power of a grating is given as resolving power $= nN$

Where n is order and N is total number of line of grating. Thus, resolving power of a grating is proportional to the total number of liens on it.

(a) total number of lines $= W \times N = 1 \times 5000 = 5000$

(b) total number of lines $= 1.5 \times 4000 = 6000$

(c) total number of lines $= 2 \times 2400 = 4800$

(d) total number of lines $= 3 \times 1500 = 4500$

Hence, total numbers of lines are maximum for option (b).

Ans. 11: (c)

Solution: In a grating the maxima is given as $(b+e)\sin\theta = n\lambda \Rightarrow \frac{n\lambda}{b+e} = \sin\theta \le 1$

$\Rightarrow \frac{n}{b+e} \le \frac{1}{\lambda} \Rightarrow nN \le \frac{1}{\lambda}$ where $N = \frac{1}{b+e}$ is number of lines per centimeter (cm) for visible and ultraviolet region we can take λ as $1000\,\overset{0}{A}$ and n as 10

$$N \le \frac{1}{1000 \times 10^{-8} \times 10} \Rightarrow N \le 10,000$$

Ans. 12: (b)

Solution: If a is width of a slit the diffraction pattern is given as

$$b\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n\lambda}{b}$$

If θ is very small then $\theta = \frac{n\lambda}{b}$

Here, $n = 1, \lambda = 600 \times 10^{-9}\ m, b = 0.2 \times 10^{-3}$

So, $\theta = \frac{1 \times 600 \times 10^{-9}}{0.2 \times 10^{-3}} = 3 \times 10^{-3}\ rad$

Ans. 13: (b)

Solution: The diffraction pattern is given as

$$b\sin\theta = n\lambda$$

Since the wavelength of violet light is less than that of red light so if red light is replaced by violet light the diffraction pattern becomes narrower.

Ans. 14: (a)

Solution: The resolving power of grating is given $\frac{\lambda}{\Delta\lambda} = nN$

Here, $n = 2$ So, $N = \frac{\lambda}{2\Delta\lambda} \Rightarrow N = \frac{5880}{2\times 6} = 491$ $\because \Delta\lambda = 5886 - 5880 = 6$

Ans. 15: (c)

Solution: Minima $= n\lambda$

$\Rightarrow$ $n_1\lambda_1 = n_2\lambda_2 \Rightarrow \lambda_1 = 3\lambda_2$

MSQ (Multiple Select Questions)

Ans. 16: (b) and (d)

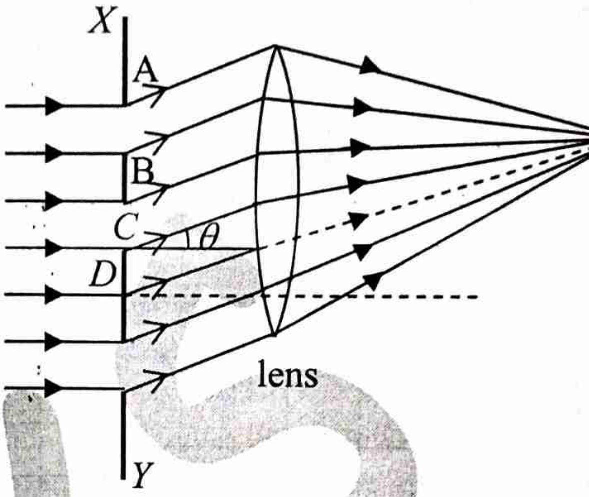
Solution: Let the width of each slit is a and b is opaque spacing between two consecutive slits.

Then the nth order principal maxima is given as

$$(b+e)\sin\theta = n\lambda$$

If the number of slits is decreased keeping the other terms constant then fringes becomes

(i) less brighter and (ii) broader



Ans. 17: (a), (b), (c) and (d)

Solution: The diffraction can take place in all above waves.

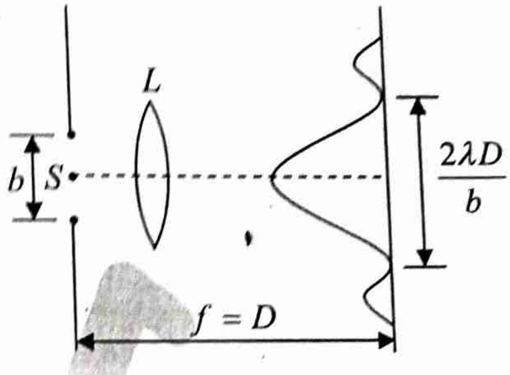
Ans. 18: (a), (c) and (d)

Solution: Since λ (Red-light) is more than yellow light. Thus the fist minima appears at highe angle as a result the width of the central maxima increases. But intensity of centra maxima remain same as incident intensity is unchanged.

NAT (Numerical Answer Type)

Ans. 19: 2.945

Solution: width of central maxima $=\dfrac{2\lambda D}{b}$, during diffraction.



or $2\times10^{-3}=\dfrac{2\times\left(5.89\times10^{-7}\right)\times0.5}{b}$

or $b=2.945\times10^{-4}\ m$.

Ans. 20: 988

Solution: Here, $\lambda_1=589.59\times10^{-6}\ m$, $\lambda_2=599.59\times10^{-6}\ m$

so, $\Delta\lambda=\lambda_1-\lambda_2=589.59\times10^{-6}-588.99\times10^{-6}=0.597\times10^{-6}\ m$

The resolving power of grating is given as $\dfrac{\lambda}{\Delta\lambda}=nN$

where n is the order of diffraction and N is the number of slits in grating.

Here $n=1$

By Equation (i) and (ii) $N=\dfrac{\lambda}{\Delta\lambda n}=\dfrac{589.59\times10^{-6}}{0.597\times10^{-6}}$

Therefore, the minimum number of lines required for just resolution in the first order in 988.

Ans. 21: 0.025

Solution: The grating can resolve, if $\dfrac{\lambda}{d\lambda}=nN$

Here $\lambda=5000\overset{0}{A}\Rightarrow\lambda=5000\times10^{-10}m$, $n=2$ and $N=105$

So, $\dfrac{1}{d\lambda}=\dfrac{nN}{\lambda}\Rightarrow d\lambda=\dfrac{\lambda}{nN}=\dfrac{5000\times10^{-10}}{2\times10^{5}}\mathring{A}=0.025\mathring{A}$

Ans. 22: 6°

Solution: **Dispersive power of a grating:** The dispersive power of a grating is defined as the ratio of difference in angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between two spectral lines. It can also be defined as difference in angle of diffraction per unit change in wavelength..

The diffraction of the n^{th} order principal maxima for a wavelength λ is given as

$$(b+e)\sin\theta = n\lambda$$

Differentiating $(b+e)\cos\theta.d\theta = nd\lambda \Rightarrow \frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \Rightarrow \frac{d\theta}{d\lambda} \propto n$

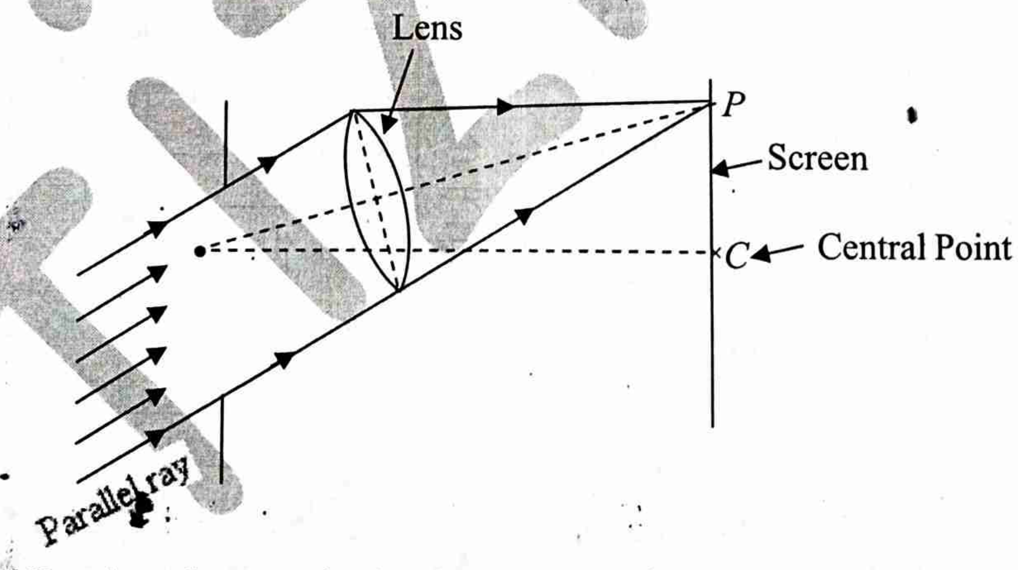
Thus, dispersive power $\propto n$

Here, $\left(\frac{d\theta}{d\lambda}\right) = An$ $\qquad A = \text{constant}$

$\left(\frac{d\theta}{d\lambda}\right)_1 = 3^{\circ}, n = 1$ and $\left(\frac{d\theta}{d\lambda}\right)_2 = \left(\frac{d\theta}{d\lambda}\right) \times \left(\frac{n_2}{n_1}\right) 3^{\circ} \times \frac{2}{1} = 6^{\circ}$

Ans. 23: 0.45

Solution:



Fraunhofer diffraction due to single slit. If b is diameter of lens the minima for diffraction pattern is given as $b\sin\theta = n\lambda$

$\Rightarrow \qquad (\text{distancce}) \propto \lambda$

$\Rightarrow \qquad \text{distance} = (0.3\ cm) \times \left(\frac{6000}{4000}\right) = 0.45\ cm$

4. Polarization of Light

Plane of Incidence

Incident ray, reflected ray, refracted ray and normal to incidence forms a plane that plane is plane of incidence.

Plane of Polarization

The plane passing through the direction of propagation and containing no vibration is called the "Plane of Polarization".

4.1 Production of Plane Polarized Light

Different methods of production of polarized light

(i) Polarization by reflection

(ii) Polarization by refraction

(iii) Polarization by selective absorption

(iv) Polarization by double refraction

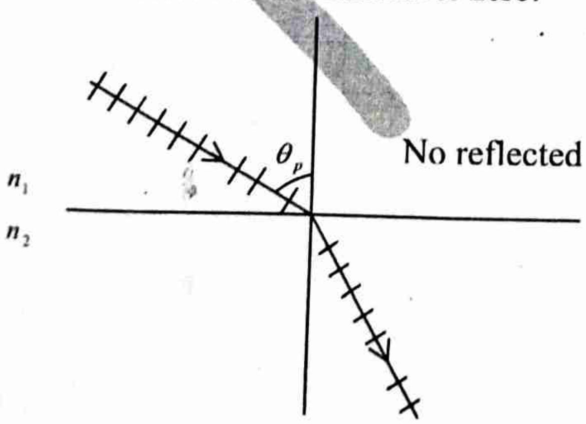
(v) Polarization by scattering

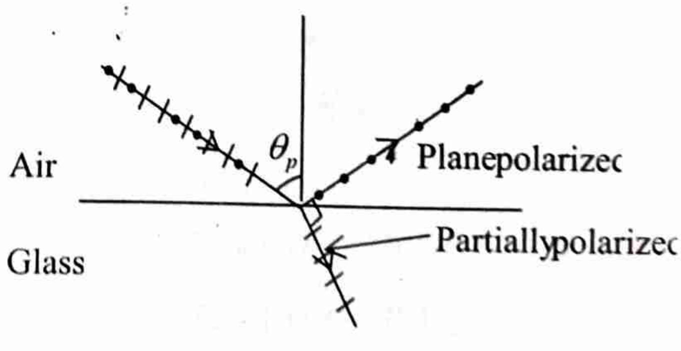
4.1.1 Polarization by Reflection

If a linearly polarized wave (Electric vector associated with the incident wave lies in the plane of incidence) is incident on the interface of two dielectrics with the angle of incidence equal to θ. If the angle of incidence θ is such that

$$\theta = \theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

then the reflection coefficient is zero.





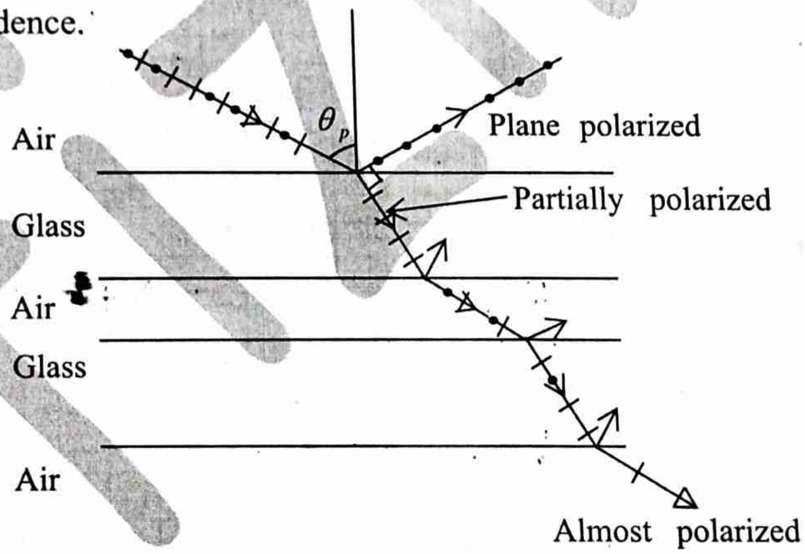
Thus if an unpolarized beam is incident with an angle of incidence equal to θ_p, the reflected beam is plane polarized whose electric vector is perpendicular to the plane of incidence.

Above equation is known as Brewster's law. The angle θ_p is known as the polarizing angle or the Brewster angle. At this angle, the reflected and the refracted rays are at right angle to each other i.e. $\theta_p + r = \frac{\pi}{2} \Rightarrow n = \frac{\sin\theta_p}{\sin r} = \frac{\sin\theta_p}{\cos\theta_p} \Rightarrow \tan\theta_p = n$

For the air-glass interface, $n_1 = 1$ and $n_2 \approx 1.5$ giving $\theta_p \approx 57^0$.

4.1.2 Polarization by Refraction

If an unpolarized beam is incident with an angle of incidence equal to θ_p, the reflected beam is plane polarized whose electric vector is perpendicular to the plane of incidence. The transmitted beam is partially polarized and if this beam is made to undergo several reflections, then the emergent beam is almost plane polarized with its electric vector in the plane of incidence.

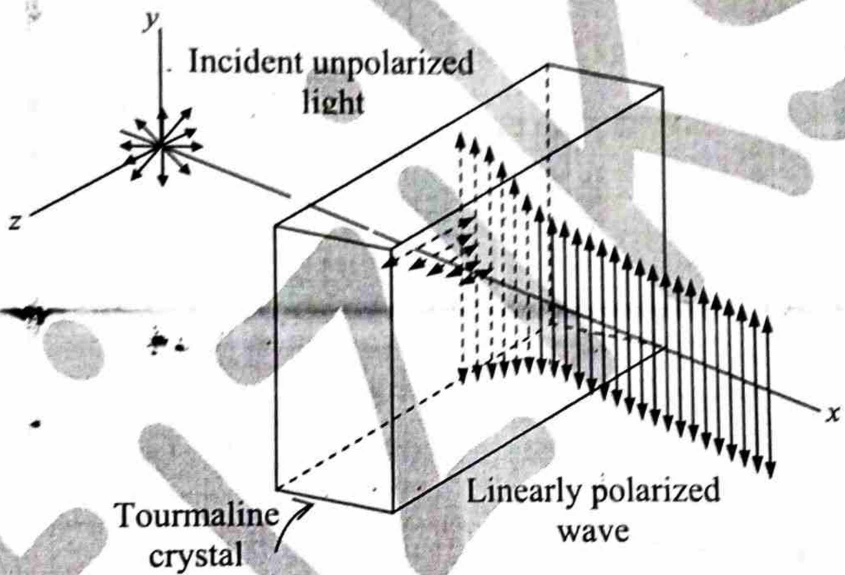


If I_p and I_s be the intensity of the parallel and perpendicular component in refracted light, then the degree of polarization is given by $P = \frac{I_p - I_s}{I_p + I_s} = \frac{m}{m + \left(\frac{2n}{1-n^2}\right)^2}$

where m is the number of plate and n is the refractive index.

4.1.3 Polarization by selective absorption

A simple method for eliminating one of the beams is through selective absorption; this property of selective absorption is known as dichroism. A crystal such as tourmaline has different coefficients of absorption for the two linearly polarized beams into which the incident beam splits up. Consequently, one of the beams gets absorbed quickly, and the other component passes through without much attenuation. Thus, if an unpolarized beam is passed through a tourmaline crystal, the emergent beam will be linearly polarized

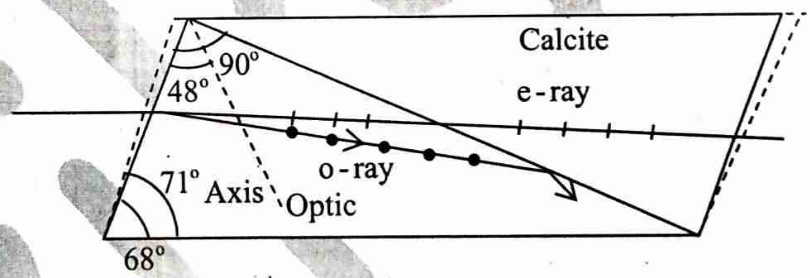


4.1.4 Polarization by Double Refraction

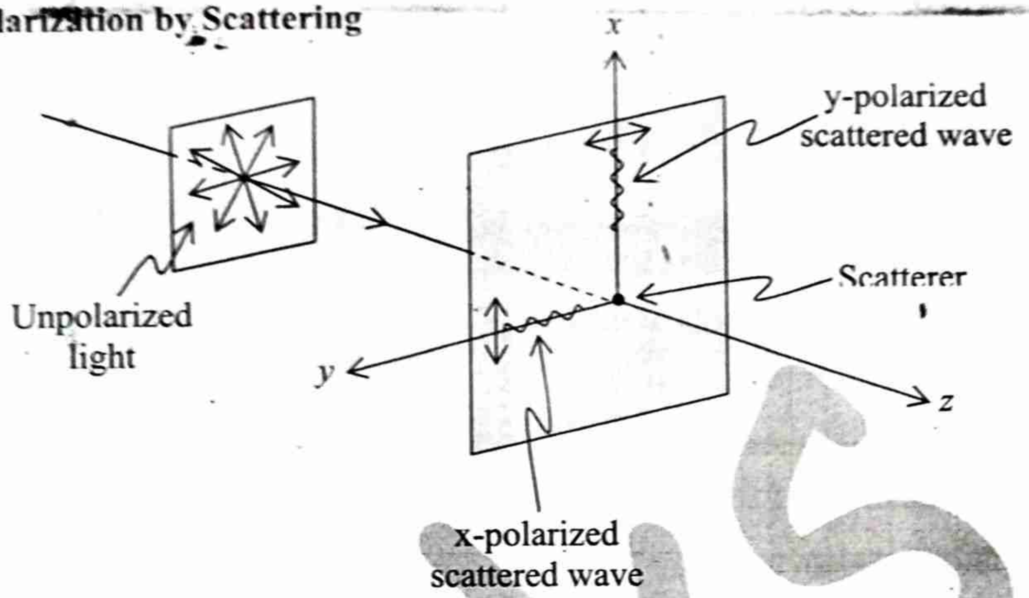
When a ray of unpolarized light passed through doubly refracting crystal (calcite or quartz), it split up into two refracted rays. One of the ray obeys the ordinary laws of refraction i.e. (n remains constant) and it is called 'ordinary ray' (o-ray). The other behaves in an extra ordinary way (i.e. n varies) and called 'extraordinary ray' (e-ray).

It is found that both the ordinary and extraordinary rays are plane-polarized having vibration perpendicular to each other.

If one can sandwich a layer of a material whose refractive index lies between the two, then for one of the beams, the incidence will be at a rarer medium and for the other it will be at a denser medium. This principle is used in a Nicol prism which consists of a calcite crystal cut in such a way that for the beam, for which the sandwiched material is a rarer medium, the angle of incidence is greater than the critical angle. Thus this particular beam will be eliminated by total internal reflection. Following figure shows a properly cut calcite crystal in which a layer of Canada balsam has been introduced so that the ordinary ray undergoes total internal reflection. The extraordinary component passes through, and the beam emerging from the crystal is linearly polarized.



4.1.5 Polarization by Scattering



When an unpolarized beam moving in z-axis, strike a gas atom than the electric field in unpolarized beam sets the electric charges in the gas atom in vibration. Since the E-field is in xy plane, therefore, the vibration will take place in xy plane only. These vibration can resolve in x and y direction and can said that two dipoles oscillating one in x and other in y-direction with the frequency of incident light. Since an oscillating dipole does not radiate in the direction of its own length.

Therefore, observer along x-axis will receive light component vibrating only in y-direction and along y-axis the component will vibrate only x axis. Hence observer will receive Plane Polarized light.

Blue colour of sky

Lord Rayleigh Formula: Intensity of light scattered from firie particles (dimension $\leq \lambda$) varies inversely as the fourth power of λ. i.e. $I \propto \frac{1}{\lambda^4}$.

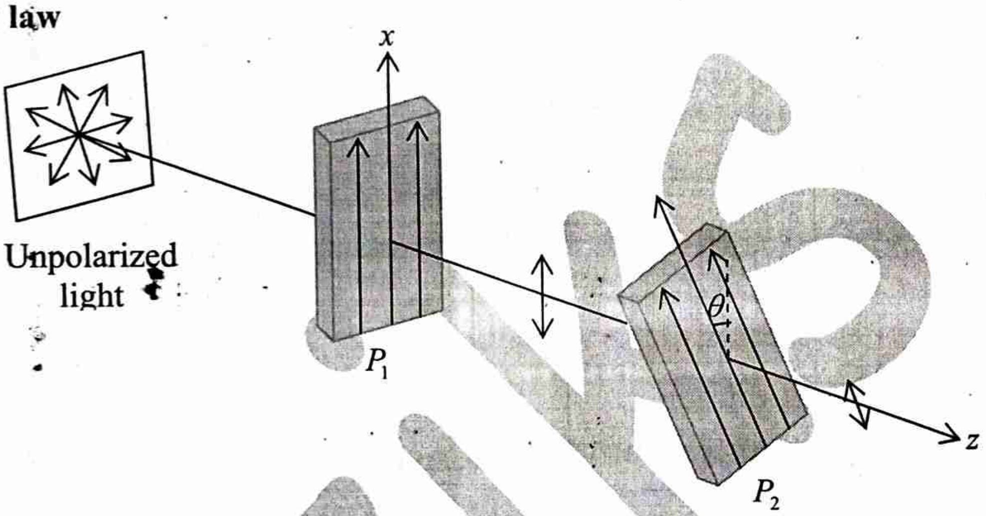
Scattered intensity of blue light is more than red $\because \lambda_{blue} < \lambda_{red}$.

Why blue scattered more: Natural frequency of bound electron in gas lies in ultra-violet region. Blue light closer to natural frequency than red light, therefore, blue is more effective in causing the electron to oscillate and thus it get more effectively scattered than red.

Red colour of sunrise and sunset

The path of the light through the atmosphere at sunrise and sunset is greatest. Since the violet & blue light is largely scattered and get removed and what we see is rest having red component.

4.2 Malus' law



An unpolarized light beam gets polarized after passing through the Polaroid P_1 which has a pass axis parallel to the x axis. When this x-polarized light beam incident on the second Polaroid P_2 whose pass axis makes an angle θ with the x axis, than the intensity of the emerging beam will vary as

$$I = I_0 \cos^2 \theta$$

where I_0 represents the intensity of the emergent beam when the pass axis of P_2 is also along the x axis (i.e., when $\theta = 0$), above equation known as Malus' law.

Thus, if a linearly polarized beam is incident on a Polaroid and if the Polaroid is rotated about the z axis, then the intensity of the emergent wave will vary according to the above law.

4.3 Superposition of Two Disturbances and Production of Polarized Wave

4.3.1 Superposition of Two Waves with Parallel Electric Field

Let us consider the propagation of two linearly polarized electromagnetic waves (both propagating along the z axis) with their electric vectors oscillating along the x axis. The electric fields associated with the waves can be written in the form

$$E_1 = \hat{x}a_1\cos(kz-\omega t+\theta_1) \quad (1)$$

$$E_2 = \hat{x}a_2\cos(kz-\omega t+\theta_2) \quad (2)$$

where a_1 and a_2 represent the amplitudes of the waves, $\hat{x}$ represents the unit vector along the x axis, and θ_1 and θ_2 are phase constants. The resultant of these two waves is given by

$$E = E_1 + E_2 \quad (3)$$

which can always be written in the form

$$E = \hat{x}a\cos(kz-\omega t+\theta) \quad (4)$$

where $$a = \left[a_1^2 + a_2^2 + 2a_1a_2\cos(\theta_1-\theta_2)\right]^{\frac{1}{2}} \quad (5)$$

represents the amplitude of the wave. Equation (4) tells us that the resultant is also a linearly polarized wave with its electric vector oscillating along the same axis.

4.3.2 Superposition of Two Waves with Mutually Perpendicular Electric field

We next consider the superposition of two linearly polarized electromagnetic waves (both propagating along the z axis) but with their electric vectors oscillating along two mutually perpendicular directions. Thus, we may have

$$E_1 = \hat{x}a_1\cos(kz-\omega t) \quad (6)$$

$$E_2 = \hat{y}a_2\cos(kz-\omega t+\theta) \quad (7)$$

For $\theta = n\pi$, the resultant will also be a linearly polarized wave with its electric vector oscillating along a direction making a certain angle with the x axis; this angle will depend on the relative values of a_1 and a_2.

To find the state of polarization of the resultant field, we consider the time variation of the resultant electric field at an arbitrary plane perpendicular to the z axis which we may, without any loss of generality, assume to be $z = 0$.

If E_x and E_y represent the x and y components of the resultant field $E = (E_1 + E_2)$, then

$$E_x = a_1 \cos \omega t \quad (8)$$

and

$$E_y = a_2 \cos(\omega t - \theta) \quad (9)$$

where we have used Equations (6) and (7) with $z = 0$.

For $\theta = n\pi$ the above equations simplify to

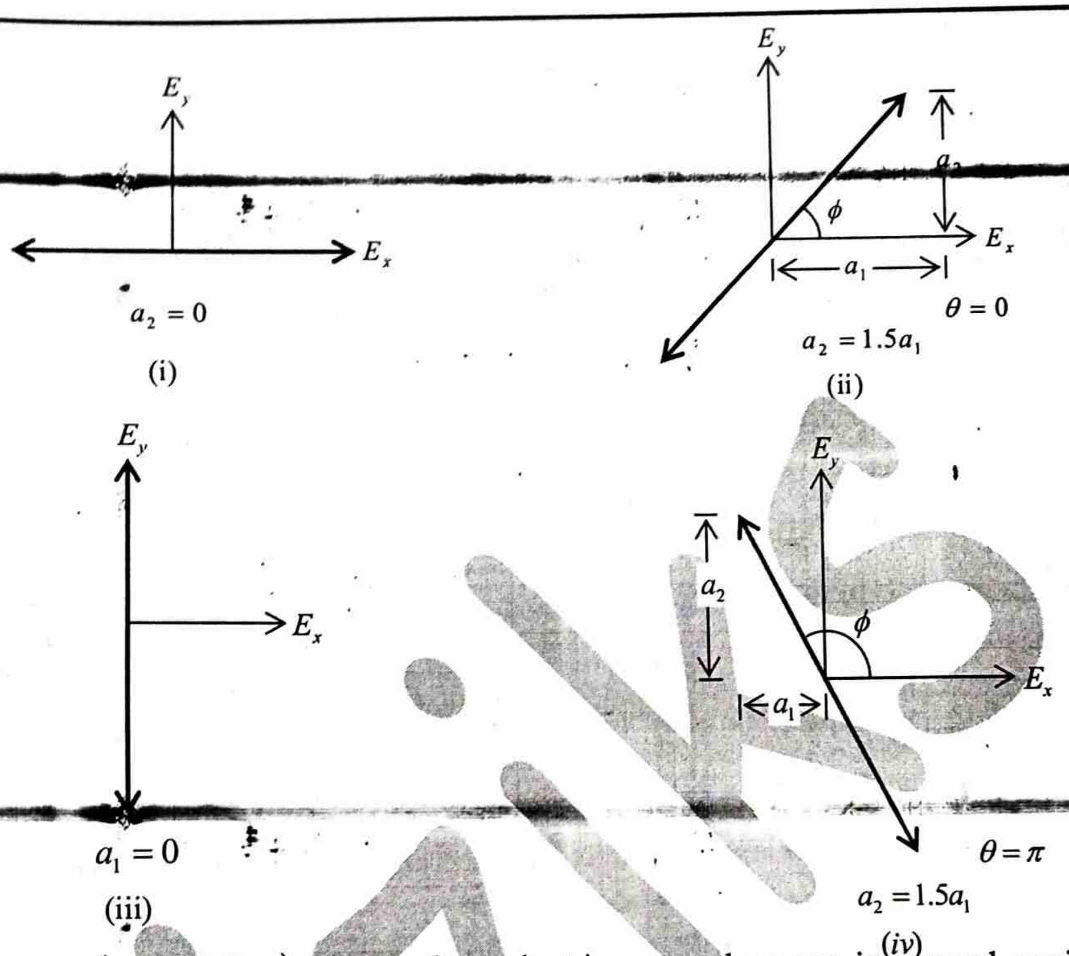
$$E_x = a_1 \cos \omega t \quad \text{and} \quad E_y = (-1)^n a_2 \cos \omega t \quad (10)$$

from which we obtain $\frac{Ey}{E_x} = \pm \frac{a_2}{a_1}$ (independent of t) (11)

where the upper and lower signs correspond to n even and n odd, respectively. In the $E_x E_y$ plane, Eq. (11) represents a straight line; the angle ϕ that this line makes with the E_x axis depends on the ratio $\frac{a_2}{a_1}$. In fact $\phi = \tan^{-1}\left(\pm \frac{a_2}{a_1}\right)$ (12)

The condition $\theta = n\pi$ implies that the two vibrations are either in phase $(n = 0,2,4....)$ or out of phase $(n = 1,3,5,...)$. Thus, the superposition of two linearly polarized electromagnetic waves with their electric fields at right angles to each other and oscillating in phase is again a linearly polarized wave with its electric vector, in general, oscillating in a direction which is different from the fields of either of the two waves.

Following figures shows the plot of the resultant field corresponding to Eq. (10) for various values of $\frac{a_2}{a_1}$. The tip of the electric vector oscillates (with angular frequency ω) along the thick lines shown in the figure. The equation of the straight line is given by Eq. (11).



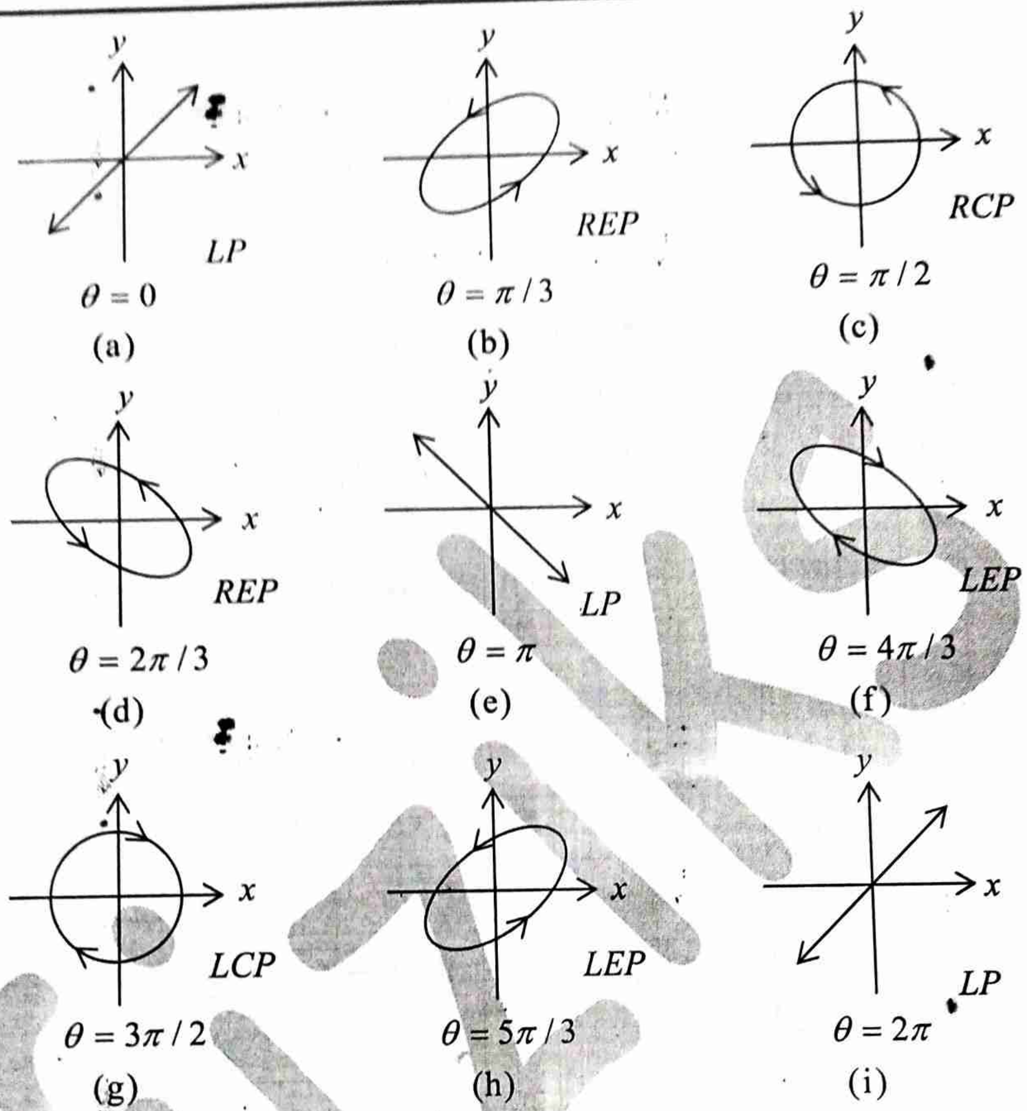
For $\theta \neq n\pi\,(n = 0,1,2,...)$, the resultant electric vector does not, in general, oscillate along a straight line.

We first consider the simple case corresponding to $\theta = \frac{\pi}{2}$ with $a_1 = a_2$. Thus,

$E_x = a_1 \cos\omega t$ (13) and $E_y = a_1 \cos\omega t$ (14)

If we plot the time variation of the resultant electric vectors whose x and y components are given by Eqs. (13) and (14), we find that the tip of the electric vector rotates on the circumference of a circle (of radius a_1) in the counterclockwise direction as shown in Fig. (c) below, and the propagation is in the $+z$ direction which is coming out of the page. Such a wave is known as a right circularly polarized wave (usually abbreviated as a RCP wave). That the tip of the resultant electric vector should lie on the circumference of circle is also obvious from the fact that

$$E_x^2 + E_y^2 = a_1^2 \qquad \text{(Independent of } t\text{)}$$



For $\theta = \frac{3\pi}{2}$, $E_x = a_1 \cos \omega t$ (15) and $E_y = -a_1 \sin \omega t$ (16)

which would also represent a circularly polarized wave; however, the electric vector will rotate in the clockwise direction [Fig. (g)]. Such a wave is known as a left circularly polarized wave (usually abbreviated as a LCP wave). For $\theta \neq \frac{m\pi}{2} (m = 0,1,2,...)$, the tip of the electric vector rotates on the circumference of an ellipse. As can be seen from the figure, this ellipse will degenerate into a straight line or a circle when θ becomes an even or an odd multiple of $\frac{\pi}{2}$. In general, when $a_1 \neq a_2$, one obtains an elliptically polarized wave which degenerates into a straight line for $\theta = 0, \pi, 2\pi, ...$ etc.

4.4 The Phenomenon of Double Refraction

When an unpolarized light beam is incident normally on a calcite crystal, it would in general, split up into two linearly polarized beams as shown in Fig. (a). The beam which travels undeviated is known as the ordinary ray (usually abbreviated as the o – ray) and obeys Snell's laws of refraction. On the other hand, the second beam, which in general does not obey Snell's laws, is known as the extraordinary ray (usually abbreviated as the e – ray).

The appearance of two beams is due to the phenomenon of double refraction, and a crystal such as calcite is usually referred to as a double refracting crystal. If we put a Polaroid PP' behind the calcite crystal and rotate the Polaroid (about NN'), then for two positions of the Polaroid (when the pass axis is perpendicular to the plane of the paper) the e - ray will be completely blocked and only the o-ray will pass through.

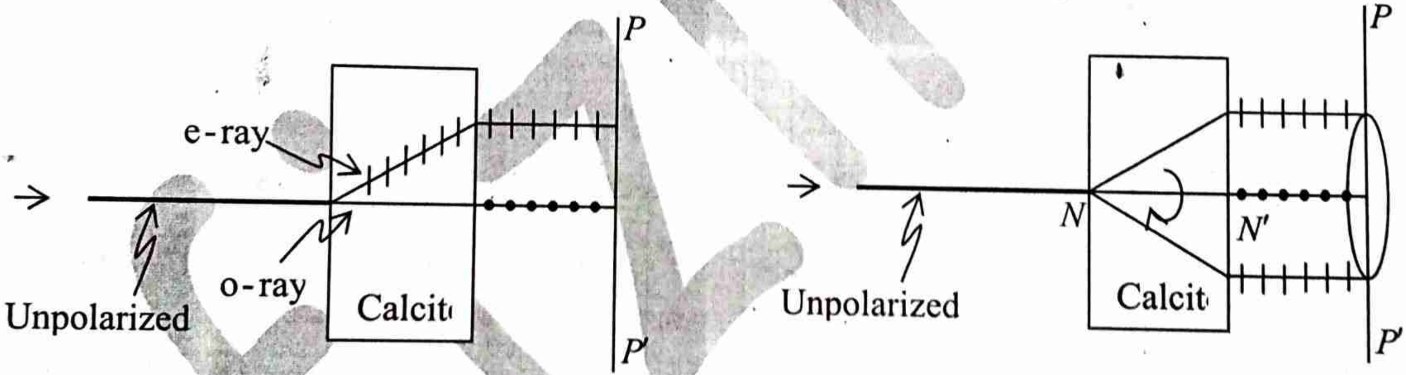


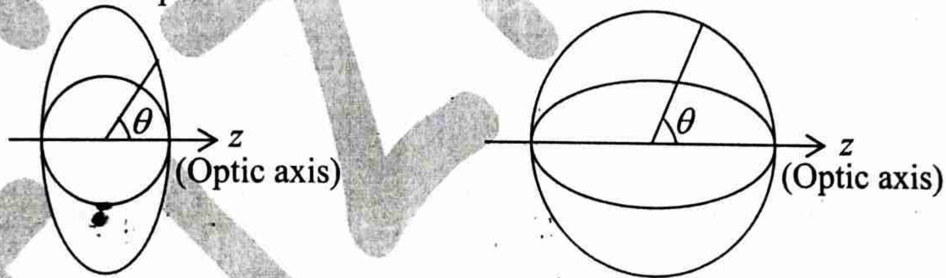
Fig (a) When an unpolarized light beam is incident normally on a calcite crystal, it would in general, split up into two linearly polarized beams. (b) If we rotate the crystal about NN' then the e-ray will rotate about NN'.

On the other hand, when the pass axis of the Polaroid is in the plane of the paper (i.e., along the line PP'), then the o - ray will be completely blocked and only the e-ray will pass through. Further, if we rotate the crystal about NN' then the e -ray will rotate about the axis [see Fig. (b)].

The velocity of the ordinary ray is the same in all directions, the velocity of the extraordinary ray is different in different directions; a substance (such as as calcite, quartz) which exhibits different properties in different directions is called an anisotropic substance. Along a particular direction (fixed in the crystal), the two velocities are equal; this direction is known as the optic axis of the crystal. In a crystal such as calcite, the two rays have the same speed only along one direction (which is the optic axis); such crystals are known as uniaxial crystals. The velocities of the ordinary and the extraordinary rays are given by the following equations:

$$v_{ro} = \frac{c}{n_o} \text{ (Ordinary ray) and } \frac{1}{v_{r_e}^2} = \frac{\sin^2\theta}{\left(\frac{c}{n_e}\right)^2} + \frac{\cos^2\theta}{\left(\frac{c}{n_o}\right)^2} \text{ (extraordinary)}$$

where n_o and n_e are refractive index for O - ray and e - ray and θ is the angle that the ray makes with the optic axis; we have assumed the optic axis to be parallel to the z axis. Thus, $\frac{c}{n_o}$ and $\frac{c}{n_e}$ are the velocities of the extraordinary ray when it propagates parallel and perpendicular to the optic axis.

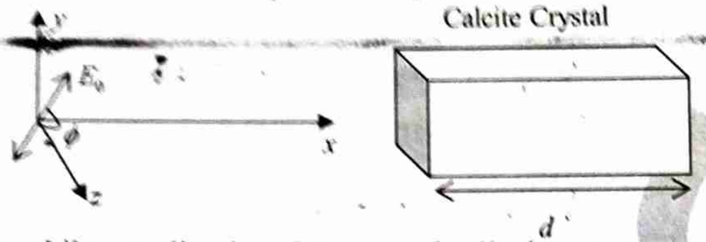


Negative crystal (Calcite) (a) Positive crystal (Quartz) (b)

Fig. (a) In a negative crystal, the ellipsoid of revolution (which corresponds to the extra ordinary ray) lies outside the sphere; the sphere corresponds to the ordinary ray. (b) In a positive crystal, the ellipsoid of revolution (which corresponds to the extraordinary ray) lies inside the sphere.

4.5 Quarter Wave Plate and Half Wave Plate

Let electric field vector (of amplitude E_0) associated with the incident linearly polarized beam makes an angle ϕ with optic axis which is parallel to z-axis and incident on calcite crystal of thickness d whose optic axis is parallel to the surface



Such beam while traveling in calcite crystal splits into two components. The z-axis whose amplitude is $E_0\cos\phi$ passes through as an extraordinary ray (e-ray) propagates with velocity c/n_e. The y-axis whose amplitude is $E_0\sin\phi$ passes through as an ordinary ray (o-ray) propagates with velocity c/n_o.

Since $n_o \neq n_e$ the two beams will propagate with different velocities, thus when they come out of the crystal, they will not be in phase. Let on the plane $x=0$, the beam is incident then

$$E_y = E_0\sin\phi\cos(kx-\omega t) \text{ and } E_z = E_0\cos\phi\cos(kx-\omega t).$$

Thus at $x=0$, we have $E_y = E_0\sin\phi\cos\omega t$ and $E_z = E_0\cos\phi\cos\omega t$.

Inside crystal the two components will be

$$E_y = E_0\sin\phi\cos(n_o kx-\omega t) \text{ and } E_z = E_0\cos\phi\cos(n_e kx-\omega t).$$

If thickness of the crystal is d then on emerging surface we have

$$E_y = E_0\sin\phi\cos(n_o kd-\omega t) \text{ and } E_z = E_0\cos\phi\cos(n_e kd-\omega t).$$

Thus the phase difference between e-ray and o-ray is $\delta = kd(n_o - n_e)$.

If phase difference is $\delta = \pi/2$, then $d = \dfrac{\lambda}{4(n_o - n_e)}$ (Quarter Wave Plate)

If phase difference is $\delta = \pi$, then $d = \dfrac{\lambda}{2(n_o - n_e)}$ (Half Wave Plate)

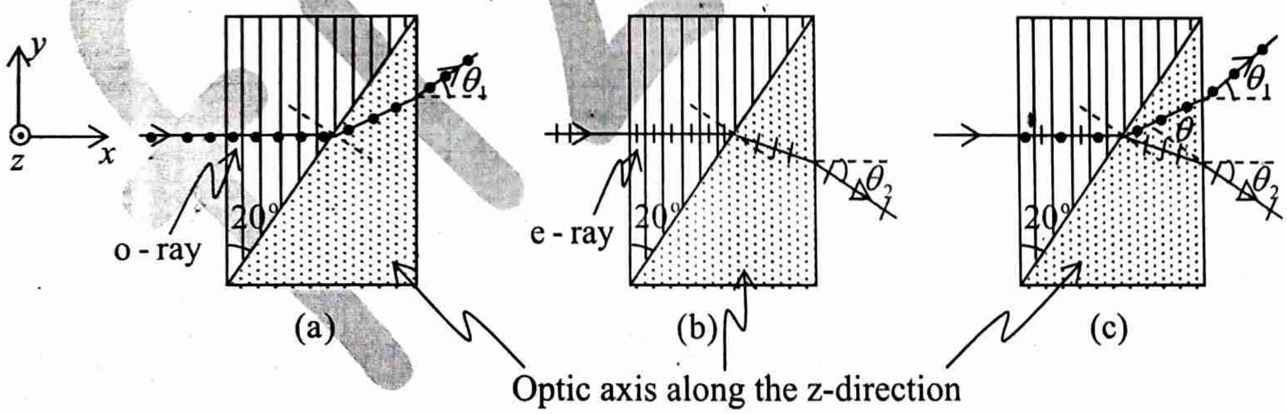
4.6 Wollaston Prism

A Wollaston prism is used to produce two linearly polarized beams. It consists of two similar prisms (calcite) with the optic axis of the first prism parallel to the surface and the optic axis of the second prism parallel to the edge of the prism as shown below. Let us first consider the incidence of a z polarized beam as shown in Fig. (a). The beam will propagate as an $o-$ ray in the first prism (because the vibrations are perpendicular to the optic axis) and will see the refractive index n_0. When this beam enters the second prism, it will become an e-ray and will see the refractive index n_e. For calcite $n_o > n_e$ and the ray will bend away from the normal. Since the optic axis is normal to the plane of paper, the refracted ray will obey Snell's laws, and the angle of refraction will be given by

$$n_0 \sin 20^o = n_e \sin r_1$$

where we have assumed the angle of the prism to be 20^o. Assuming $n_o \approx 1.658$ and $n_e \approx 1.486$, we readily get $r_1 \approx 22.43^o$

Thus the angle of incidence at the second surface is $i_1 = 22.43^o - 20^o = 2.43^o$. The output angle θ_1 is given by $n_e \sin 2.43^o = \sin\theta_1 \Rightarrow \theta_1 = 3.61^o$.



We next consider the incidence of a y - polarized beam as shown in Fig. (b). The beam will propagate as an e-ray in the first prism and as an o-ray in the second prism. The angle of refraction is now given by

$$n_e \sin 20 = n_o \sin r_2 \Rightarrow r_2 \approx 17.85^o$$

Thus the angle of incidence at the second interface is

$$\therefore \quad i_2 = 20^o - 17.85^o = 2.15^o$$

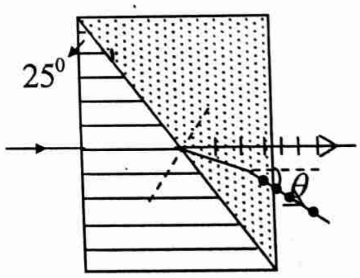
The output angle θ_2 is given by

$$n_o \sin 2.15^o = \sin\theta_2 \Rightarrow \theta_2 \approx 3.57^o$$

Thus, if an unpolarized beam is incident on the Wollaston prism, the angular separation between the two orthogonally polarized beams is $\theta = \theta_1 + \theta_2 \approx 7.18^o$.

4.7 Rochon Prism

We next consider the Rochon prism which consists of two similar prisms of (say) calcite; the optic axis of the first prism is normal to the face of the prism while the optic axis of the second prism is parallel to the edge as shown in figure. Now, in the first prism both beams will see the same refractive index n_o; this follows from the fact that the ordinary and extraordinary waves travel with the same velocity $\left(\frac{c}{n_o}\right)$ along the optic axis of the crystal. When the beam enters the second crystal, the ordinary ray (whose D is normal to the optic axis) will see the same refractive index and go undeviated as shown in figure. On the other hand, the extraordinary ray (whose D is along the optic axis) will see the refractive index n_e and will bend away from the normal.



We assume the angle of the prism to be 25^o. The angle of refraction will be determined from

$$n_o \sin 25^o = n_e \sin r$$

Thus $\sin r = \frac{n_o}{n_e}\sin 25^o = \frac{1.658}{1.486}\times 0.423 \approx 0.472 \Rightarrow r = 28.2^o$

Therefore the angle of incidence at the second surface will be $28.2^o - 25^o = 3.2^o$. The emerging angle will be given by $\sin\theta = n_e \sin\left(3.2^o\right) \Rightarrow \theta \approx 4.8^o$