Maa Omwati Degree Collage

Hassanpur

NOTES

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CONTENTS

Basic Nuclear Properties

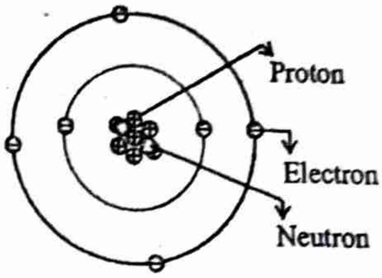
Introduction:

Rutherford first told that inside the atom there is a very small nucleus in which all the positive charge and most of the mass is localized after doing the α -ray scattering experiments. After then scienties were strating to investigate the properties of nucleus. In this chapter we will read about nuclear charge nuclear mass, size, shape, binding energy, angular momentum, magnetic moment, statistics, parity, isospin.

Structure of nuclei:

There are two types of particles inside the nucleus called nucleon.

(i) Proton : it has positive charge (p) (ii) Neutron : it is neutral (n)



Atomic number:

The number of protons inside the nucleus is called atomic number.

It is represented by 'z'.

Mass number:

The total number nucleons (proton + neutron) is called mass number. It is represented by 'A' and $A = Z + N$.

Symbolically representation of nucleus:

A nucleus is symbollically represented by $_zX^A$ like $_8O^{16}$, $_2He^4$.

Isotopes:

Nuclei with the same atomic number z, but different mass number A are called isotopes like $_1H^1$, $_1H^2$, $_1H^3$ are called isotops.

Isobars:

Nuclei with same mass number A but different atomic number z are called isobers like $_8O^{16}$ and $_7N^{16}$ are isobar.

Isotones:

Nuclei with same number of neutrons N are called isotones like $_{11}Na^{23}$ and $_6C^{13}$ are isotones.

Mirror nuclei:

A pair of isobaric nuclei are known as mirror nuclei where in the nucleus the proton number z and the neutron number N are interchanged and differ by one unit is called Mirror nuclei, like $^{11}C_6$ and $^{11}B_5$ are mirror nucleus.

Binding energy of nucleus:

Inside the nucleus protons and neutrons are strongly bound. To separate neutrons and protons from each other we need few MeV energy which is called the binding energy E_B of the nucleus. But the question is what is the source of this energy?

According to mass energy equivalence of special relativity, the energy equivalent corresponding to complete conversion of a mass m into energy is mc^2, where 'c' is the velocity of light in free space. In forming a nucleus out of constituent particles, a fraction of the total mass of the constituents disappears and produce energy equal to E_B.

If ΔM be the amount of mass disappeared, then the energy, $E_B = \Delta Mc^2$

If M_p, M_n be the masses of proton and neutron respectievely .

$$\Delta M = ZM_p + NM_n + ZMe - M(A,Z)$$

where, $M(A,Z)$ is the mass of the atom of mass number A and atomic number Z.

$$E_B = \left[ZM_p + NM_n + ZMe - M(A,Z)\right]c^2$$

$$= \left[ZM_p + NM_n + ZMe - M(nuc) - ZMe\right]c^2$$

$$= \left[ZM_p + NM_n - M_{nuc}\right]c^2$$

$$\boxed{E_B = \left[ZM_p + NM_n - M_{nuc}\right]c^2}$$

Binding fraction curve:

Binding fraction f_B is the ratio of binding energy to mass number.

$$\therefore f_B = \frac{E_B}{A} = \frac{ZM_p + NM_n - M_{nuc}}{A}$$

$$\boxed{f_B = \frac{ZM_P + NMn - M_{nuc}}{A}}$$

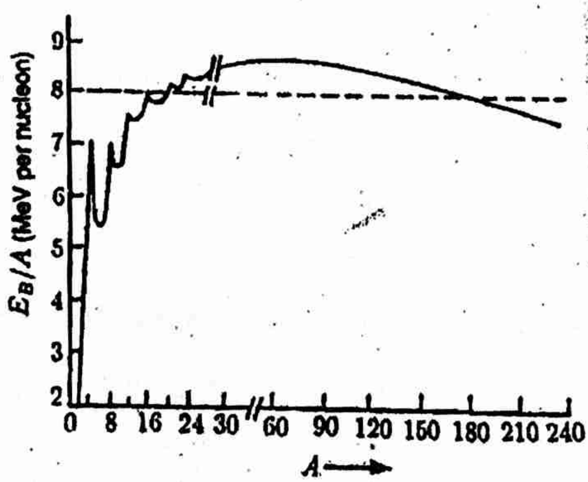


Figure: **Binding fraction curve**

Observation from graph:

1. f_B is very small for very light nuclei and goes on increasing rapidly with increasing A and reaches a value ~ 8 MeV/nucleon for the mass number A ~ 20. Thereafter the rise of the curve is much slower, reaching a maximum value of 8.7 MeV per nucleon for A = 56. If A is increased still further, the curve decreases slowly.
2. The variation in f_B is very slight in the mass number range $20 < A < 180$ and in this region f_B may be considered to be virtually constant with a mean value ~ 8.5 MeV/nucleon.
3. For $A > 180$, that is, for heavy nuclei, the f_B-value decreases monotonically with increasing A and it is ~ 7.5 MeV/nucleon for the heaviest nuclei.
4. A rapid fluctuation in f_B is noted for very light nuclei with peaks in the curve of this region, corresponding to even-even nuclei, such as ^4He, ^8Be, ^{12}C, ^{16}O i.e., with mass number A = 4n, where n = 1, 2, 3, ... Peaks in the curve are also seen at Z or N equal to 20, 28, 50, 82, 126. These are called magic numbers.

Mass defact:

The difference between the measured atomic mass $M(A,Z)$ and the mass number A of a nucleus is called the mass defect, $\Delta M'$

$$\therefore \quad \boxed{\Delta M' = M(A,Z) - A}$$

for ${}_8O^{16}$ $\quad \Delta M' = 15.994915 - 16 = -0.005085u$

Packing fraction:

The packing fraction f is defined as the mass defect per nucleon in the nucleus.

$$f = \frac{\Delta M'}{A} = \frac{M(A,Z) - A}{A}$$

$$\Rightarrow \quad \boxed{M(A,Z) = A(1+f)}$$

Nuclear size:

From experimental result we can say that the mass density of the nucleus almost remain constant.

$$\rho_m = \frac{A}{V} = \text{constant}$$

$$\therefore \quad V \propto A$$

$$\Rightarrow \quad \frac{4}{3}\pi R^3 \propto A$$

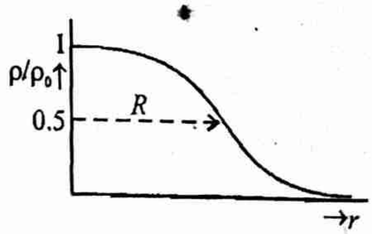
$$\Rightarrow \quad R \propto A^{1/3}$$

$$\Rightarrow \quad \boxed{R = r_0 A^{1/3}} \quad r_0 = (1.1 - 1.5) \times 10^{-15} m$$

Charge distribution of nucleus:

If $\rho(x,y,z)$ be the volume density of charge of the nucleus with its centre of mass at the origin so that $r^2 = x^2 + y^2 + z^2$. Also the charge density can be express as

$$\boxed{\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}}$$



where R is the radius of nucleus a is a constant.

The quadrupole moment Q is defined as

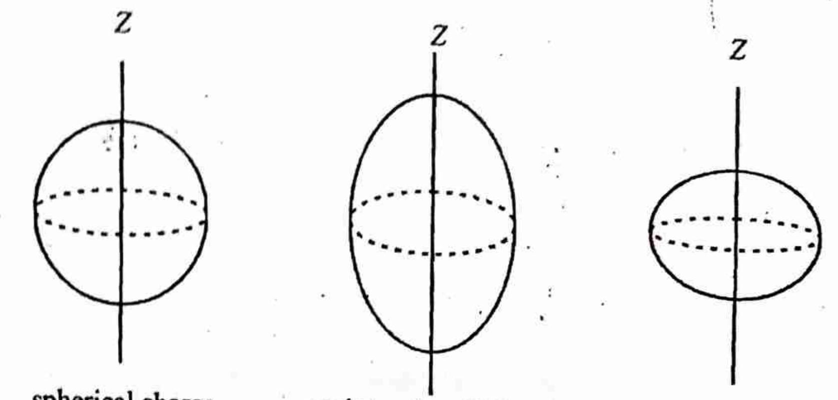
$$\boxed{Q = \frac{1}{e}\int (3z^2 - r^2)\rho dV}$$

where the integration is carried out over the entire volume of the nucleus, e is the the charge of proton, the nucleus is assumed to have a symmetry axis along z. The unit of Q is barn

$$1 \text{ barn} = 10^{-28} \text{ m}^2.$$

Special cases:

1. For spherically symmetric nucleus $Q = 0$
2. For shape of prolate spheroid (the charge distribution streched in the Z direction) $Q > 0$ (positive)
3. For shape of oblate spheroid (the charge distribution is streched perpendicular, then the quadrupole moment $Q < 0$ (negative)



spherical charge distribution Q = 0 | prolate spheroid Q > 0 | oblate spherical Q < 0

Spin of nuclei:

The spin nucleus is the resultant spins of its constituent nucleons – protons and neutrons and have spin +½. The orienation of spin vector, according to wave mechanics is such that the components of spin can have its axis either parallel or antiparallel to a specific to a specific direction with spin angular momentum $\frac{1}{2}\hbar$ or $-\frac{1}{2}\hbar$ respectively.

In addition to the spin angular momentum, the protons and neutrons in the nucleus have orbital angular momentum such that its magnitude in specific z-direction is an integral multiple $\hbar$. Thus, intrinsic angular momentum of a nucleus is a vector I such that

$$I = \sum_n \ell n + \sum_s \ell s = L + S$$

The magnitude of the total angular momentum vector I is given by

$$\bar{I} = \sqrt{I \times (I+1)}\,\hbar$$

$\bar{I} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \ldots\ldots$ for odd A type nuclei

$I = 0, 1, 2, 3 \ldots\ldots$ for even A type nuclei.

Parity of nuclei:

Parity is the property of a wave function representing a quantum mechanical nuclear state, which may or may not change its sign on inversion of the space coordinates from (x, y, z) to $(-x, -y, -z)$ through origin.

$\psi(-x,-y,-z) = \psi(x,y,z)$ then parity is even.

$\psi(-x,-y,-z) = -\psi(x,y,z)$ then parity is odd.

Nuclear forces:

We know that inside the nucleus there are proton and neutron and protons repel each other strongly and they shuld fly apart but it does not happend this tells us that there is some kind of very strong attractive force. These force are n–n, p–p and n–p force. These force have following properties.

(i) They are short range forces
(ii) They are charge-independent
(iii) They are strongest known forces in nature
(iv) They are radialy saturated by the surrounding nucleons
(v) They are spin-dependent
(vi) The force is non-central.

SOLVED PROBLEMS

1. The mean momentum of a nucleon in a nucleus with mass number A varies as: **[GATE 2000]**
(a) A (b) A^2 (c) $A^{-2/3}$ (d) $A^{-1/3}$

Soln. We know that,

$$\Delta x\Delta p = \hbar \Rightarrow \Delta p = \frac{\hbar}{\Delta x}$$

Here, $\Delta x = 2R$ where, R is the radius of nucleus.

$$\Delta p = \frac{\hbar}{2R}$$

We know that, $R = r_0 A^{1/3}$

$$\therefore \quad \Delta p = \frac{\hbar}{2r_0} A^{-1/3}$$

Therefore, we ean say mean momentum of a nucleon in a nucleus varies with mass number as $A^{-1/3}$.
Correct option is (d)

2. The masses of a hydrogen atom, neutron and $^{238}U_{92}$ are given by 1.0078, 1.0087 and 238.0508 respectively. The binding energy of $^{238}U_{92}$ is therefore approximately equal to (taking 1 a.m.u. = 931.64 MeV)
(a) 120 MeV (b) 1500 MeV (c) 1600 MeV (d) 1800 MeV **[GATE 2000]**

Soln. Binding energy of $^{238}U_{92}$ is

$$= ZM_P + NM_n - M_u$$
$$= 92\times1.0077 + 146\times1.0087 - 238.0508$$
$$= 239.9878 - 238.0508 = 1.937 \text{ a.m.u.} = 1804.58 \text{ MeV} \simeq 1800 \text{ MeV}$$

Correct option is (d)

3. The order of magnitude of the binding energy per nucleon in a nucleus is: **[GATE 2006]**
(a) 10^{-5} MeV (b) 10^{-3} MeV (c) 0.1 MeV (d) 10 MeV

Soln. From uncertainty principle

$$\Delta x\Delta p_x = \hbar$$

$$\Rightarrow \quad (\Delta p_x)_{\min} = \frac{\hbar}{2R} \quad \text{(where, R is nucleus radius)}$$

$$\therefore \quad (K.E.)_{\min} = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{8mR^2}$$

For stable nucleus we can say binding energy is the order of minimum kinetic energy

$$\text{Therefore, binding energy} = \frac{\hbar^2}{8M_pR^2} = \frac{(6.62\times10^{-34})^2}{8\times1.6\times10^{-27}\times1.6\times10^{-19}\times1.2\times1.2\times10^{-30}} \simeq 10 \; MeV$$

Correct option is (d)

4. An O^{16} nucleus is spherical and has a charge radius R and a volume $V \equiv \frac{4}{3}\pi R^3$. According to the empirical observations of the charge radii, the volume of the $_{54}X^{128}$ nucleus, assumed to be spherical, is
(a) 8V (b) 2V (c) 6.75V (d) 1.89V **[GATE 2008]**

Soln. We have, $V = \frac{4}{3}\pi R^3$

Also we know, $R \propto A^{1/3}$

$\therefore \quad V \propto A$

$$\frac{V_0}{V_x} = \frac{A_0}{A_x} = \frac{16}{128}$$

$$\Rightarrow \quad V_x = \frac{128}{16} V_0 = 8V$$

Correct option is (a)

5. The stable nucleus that has $\frac{1}{3}$ the radius of ^{189}Os nucleus is, **[JEST 2015]**

(a) ^{7}Li (b) ^{16}O (c) ^{4}He (d) ^{14}N

Soln. We know that, $V \propto A$ and $R \propto A^{1/3}$

$$\therefore \quad \frac{R_x}{R_{Os}} = \frac{1}{3} \Rightarrow \frac{A_x^{1/3}}{A_{Os}^{1/3}} = \frac{1}{3} \Rightarrow A_x^{1/3} = \frac{1}{3}(189)^{1/3} \Rightarrow A_x = \frac{189}{27} = 7$$

Therefore, the stable nucleus ^{7}Li has $\frac{1}{3}$ radius of ^{189}Os nucleus.

Correct option is (a)

PRACTICE SET

1. The experimentally measured nuclear charge density distribution ρ is approximated by the function:

(a) $\rho = \rho_0\left(1+e^{(r-R)/a}\right)$ (b) $\rho = \frac{\rho_0}{1+e^{(r-R)/a}}$ (c) $\rho = \rho_0 e^{-r/R}$ (d) $\rho = \frac{\rho_0}{r} e^{-r/R}$

2. The radius of a $^{64}_{29}Cu$ nucleus is measured to be 4.8×10^{-13} cm. The root-mean-square (rms) energy of a nucleon in a nucleus of atomic number A in its ground state varies as:

(a) $A^{4/3}$ (b) $A^{1/3}$ (c) $A^{-1/3}$ (d) $A^{-2/3}$

3. The ratio of radii of the nuclei He^4 & S^{32} is :

(a) $\frac{2}{1}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) $\frac{4}{1}$

4. The binding energy per nucleon is approximately constant this leads to :

(a) saturation property of nuclear forces (b) short range nature
(c) non-central nature (d) spin dependency

5. For a nucleus to possen quadrupole moment :

(a) $I=0$ (b) $I=\frac{1}{2}$ (c) $I>\frac{1}{2}$ (d) spherical shape of nucleus

I = nuclear spin

6. Number of nucleon/m³ in a nucleus is :

(a) 10^{40} (b) 10^{44} (c) 10^{50} (d) 10^{49}

7. Binding energy per nucleon is max. for

(a) He^4 (b) Ca^{40} (c) Fe^{56} (d) Pb^{206}

8. The nuclear density in kg/mm³ is:

(a) 10^{17} (b) 10^{13} (c) 10^{8} (d) 10^{10}

9. With reference to nuclear forces which of the following statements is NOT true? The nuclear forces are:

(a) short range (b) charge independent
(c) velocity dependent (d) spin independent

10. The deviation of the charge distribution of a nucleus from spherical symmetry can be estimated by measuring its:

(a) electric charge (b) electric dipole
(c) magnetic dipole moment (d) electric quadrupole moment

ANSWER KEY					
Questions	1	2	3	4	5
Option	(b)	(c)	(c)	(a)	(c)
Questions	6	7	8	9	10
Option	(b)	(c)	(c)	(c)	(d)

Nuclear Models

Introduction:

In previous chapter we have study about nucleus properties like variation of radius with mass number, charge density binding energy. But to understand the above behaviour of atomic nucleus, it is essential to know the exact nature of the forces between the nucleons. However, due to lack of knowledge of the exact internucleon force, it has not been possible to develop a satisfactory theory of nuclear structure . In absence of a comprehensive theory, attemps were made to develop nuclear models to explain the properties of nuclei. Several different models have been suggestd, each with its successes and limitations. The important models are

(a) The liquid drop model
(b) Nuclear shell model
(c) Fermi gas model
(d) Alpha model
(d) Collective model
(e) Optical model.

A. Liquid drop model:

The liquid drop model of the nucleus was first proposed by Niels Bohr and F.Kalcar in the year 1937. They observed that there exists many similarities between the drop of a liquid and a nucleus. For instance,

(i) both the liquid drop and the nucleus possess constant density,
(ii) The constant binding energy per nucleon of a nucleus is similar to the latent heat of vaporization of a liquid,
(iii) The evaporation of a drop corresponds to the radioactive properties of the nucleus, and
(iv) The condensation of drops bears resemblance with the formation of compound nucleus, etc.

- According to this model, the nucleus is supposed to be spherical in shape in the stable state with radius $R = r_0 A^{1/3}$, just as a liquid drop is spherical due to symmetrical surface tension forces. The surface tension effects are analogous to the potential barrier effects on the surface of the nucleus.
- The density of a liquid drop is independent of the volume, as is the case with the nucleus. But whereas the nuclear density is independent of the type of nucleus, the density of a liquid does depend on its nature.
- Like the nucleons inside the nucleus, the molecules in the liquid drop interact only with their immediate neighbours.
- The non-independence of the binding energy per nucleon on the number of nucleons in the nucleus is analogous to the non-independence of the heat of vaporization of a liquid drop on the size of the drop.
- Molecules in a liquid drop evaporate from the liquid surface in raising the temperature of the liquid due to their increased energy of thermal agitation. Analogously, if high energy nuclear projectiles bombard the nucleus, a compound nucleus is formed in which the nucleons quickly share the incident energy and the emission of nucleons occurs.

- The phenomenon of nuclear fission is easily explained as the splitting of the liquid drop into two more or less equals parts if set into vibration with sufficient energy.

- **Semi-empirical binding energy or mass formula:**

C.V. Weizsacker, a German physicist, proposed the following semi-empirical formula for the nuclear binding energy B.E. (in MeV) for the nucleus (Z, A)

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$$

with the constants having the value, $a_v = 15.75, a_s = 17.80, a_c = 0.71, a_n = 22.7$ and δ=33.6 are in MeV.

(i) Volume energy:

The first term, $E_V = a_v A$, is the volume effect representing the volume energy of all nucleous. The more the total number of nucleons A, more difficult it becomes to remove an individual nucleon from the nucleus. Since the nuclear density is nearly constant, the nuclear mass is proportional to the nuclear volume, which again is proportional to R^3. But $R \propto A^{1/3} \Rightarrow R^3 \propto A$. So, the volume energy $E_v \propto A$.

$\Rightarrow \quad E_v = a_v A$

This energy corresponds to the amount of heat energy (the heat of vaporisation) required to transform a liquid to its vapour state being proportional to the mass of the liquid.

(ii) Surface energy:

The second term, $E_s = a_s A^{2/3}$, is the surface effect being similar to the surface tension in liquids, like the molecules on the surface of a liquid, the nucleons at the surface of the nucleus are not completely surrounded by other nucleons. It results in reducing the total binding energy due to nucleons on the surface. This correction due to surface energy E_s, which is proportional to the surface area of the nucleus i.e. to $4\pi R^2$ i.e. $E_s \propto R^2 \quad \Rightarrow \quad E_s \propto A^{2/3} \quad \Rightarrow \quad E_s = a_s A^{2/3}$

(iii) Coulomb energy:

The third term, E_c, is the Coulomb electrostatic repulsion between the charged particles in the nucleus. Since each charged particle repulses all the other charged particles, this term would be directly proportional to the possible number of combinations for a given proton number Z, which is Z(Z–1)/2. The energy of interaction between protons is again inversely proportional to the distance of separation R, so the energy associated with Coulomb repulsion is:

$$E_c = k\frac{Z(Z-1)}{R} = k\frac{Z(Z-1)}{r_0 A^{1/3}} = a_c \frac{Z(Z-1)}{A^{1/3}} \quad (-\text{ve quantity})$$

(iv) Asymmetry energy:

The fourth term E_a originates from the asymmetry between the number of protons and the number of neutrons in the nucleus. Nuclear data for stable nuclei indicate that for lighter nuclei, the number of protons is almost equal to that of neutrons: N = Z. As A increases, the symmetry of proton and neutron number is lost and the number of neutrons exceeds that of protons to maintain the nuclear stability. This excess of neutrons over protons, i.e. N–Z, is the measure of the asymmetry and it decreases the stability or the B.E. of medium or heavy nuclei.

So, $E_a \propto (N-Z)$, and $E_a \propto (N-Z)A \quad \Rightarrow \quad E_a = a_n \frac{(N-Z)^2}{A} = a_n \frac{(A-2Z)^2}{A}$

(v) Pairing energy:

The last term, a pure corrective term, is called pairing energy E_p.

$$E_p = \pm\frac{\delta}{A^{3/4}}$$

Z	N	A	δ	E_p
even	even	even	34	$+\delta/A^{3/4}$
even	odd	odd	0	0
odd	even	odd	0	0
odd	odd	even	35	$-\delta/A^{3/4}$

- Binding fraction, $f_B = \frac{B.E.}{A} = a_v - \frac{a_s}{A^{1/3}} - \frac{a_c Z(Z-1)}{A^{4/3}} - a_n\frac{(A-2Z)^2}{A^2} \pm \frac{\delta}{A^{7/4}}$

Mass parabola and stability of nuclei against β-decay:

- Mass of nucleus, $M(A,Z) = ZM_p + (A-Z)M_n - B.E./c^2$

$$= ZM_p + (A-Z)M_n - \frac{1}{c^2}\left[a_0 A - a_s A^{2/3} - a_c\frac{Z(Z-1)}{A^{1/3}} - a_n\frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}\right]$$

The above formula is known as the semi-empirical mass formula.

Assuming, $F_A = A(M_n - a_v + a_n) + a_s A^{2/3}$

$p = -4a_n - (M_n - M_p)$; $q = \frac{1}{A}\left(a_c A^{2/3} + 4a_n\right)$

$\Rightarrow$ $\boxed{M(A,Z) = F_A + pZ + qZ^2}$, it is equation of parabola for given A is called mass parabola.

$\Rightarrow$ $\left(\frac{\partial M}{\partial Z}\right)_A = p + 2qZ = 0$ at $Z = Z_A$, whence, we get

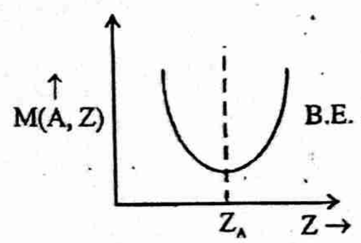
$\Rightarrow$ $Z_A = -\frac{p}{2q} = \frac{(M_n - M_p + 4a_n)A}{2\left(a_c A^{2/3} + 4a_n\right)}$ $\Rightarrow$ $Z_A = A/\left(1.98 + 0.015A^{2/3}\right)$

In most cases, the value of Z nearest to Z_A gives the actual stablest nucleus for a given A.

All isobars having mass greater than that of stable isobar. They will decay by emission of β^-, β^+ and k capture.

1. The isobar to the left of the stable one decay by β^- emission.

2. The isobar to the right of the stable one decay by β^+ emission or k-capture or by both.



Magic number : If the number of proton or neutron or both are 2, 8, 20, 28, 50, 82, 126 of a nucleous then it is very stable. These numbers are called Magic number.

B. Shell Model:

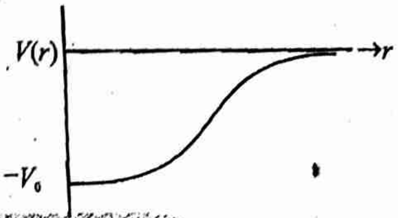
Liquid drop model can not explore why some nucleus having proton or neturon or both are 2, 8, 20, 28, 50, 82, 126 they have high binding energy than others. The scienties were starting thought that there have some arrangment like electron distribution in the atom.

Since we do not know the exact force nature inside the nucleus so we need to take a guess.

1. At first they consider potential is in the form of harmonic potential ($V(r) \propto r^2$) and they got equally spaced energy levels but they could not explains the magic numbers.

2. After then they took another guess, the potential is in the Saxon-Woods model.

$$V(r) = -\frac{V_0}{1+e^{(r-R)/\delta}}$$



where, $V_0 \simeq 50\ MeV,\ R = r_0 A^{1/3},\ \delta = 0.52\ fm$

For such a potential they got energy of the n[th] level

$$E_{n\ell} = \frac{\hbar^2 \ell(\ell+1)}{2\mu R_0^2}\left(1+\frac{12a^2}{R_0^2}\right)\frac{\hbar^2}{2\mu a^2}\left\{\frac{\left[\sqrt{1+192\ell(\ell+1)a^4}-2n-1\right]^2}{16}+\frac{4\left[\frac{\mu a^2 V_0}{\hbar^2}-\frac{4\ell(\ell+1)a^3}{R_0^3}\right]^2}{\left[\frac{\sqrt{1+192\ell(\ell+1)a^4}}{R_0^4}-2n-1\right]^2}+\frac{\mu V_0 a^2}{\hbar^2}\right\}$$

The lowest level is 1s(i.e. n = 1, l = 0) which can contain up to 2 protons or neutrons. Then comes *lp* which can contain up to a further 6 protons. This explains the first two magic numbers (2 and 8). Then there is a level 1d but this is quite close in energy to 2s so that they form the same shell. This allows a further 2 + 10 protons or neutrons giving us the next magic number of 20.

The next two levels are *1f* and *2p* which are also quite close together and allow a further 6 + 14 protons. This would suggest that the next magic number was 40 but experimentally it is known to be 50.

3. After then they add a correction term $f(r)\vec{\ell}\cdot\vec{s}$, where $f(r)$ is an fuction of r and its magnitude is negative.

Therefore, the potential become

$$V(r) = -\frac{V_0}{1+e^{(r-R)/\delta}} + f(r)\vec{\ell}\cdot\vec{s}$$

As in the case of atomic physics (J-J coupling scheme) the orbital and spin angular momentum of the nucleous combine to give a total angular momentum J which can take the values $J = \ell + \frac{1}{2}$ or $J = \ell - \frac{1}{2}$.

For $J = \ell + \frac{1}{2}$

$$\langle \ell \cdot s \rangle = \frac{\hbar^2}{2}\left[\left(\ell+\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right) - \ell(\ell+1) - \frac{3}{4}\right]$$

$$= \frac{\hbar^2}{2}\left[\ell^2 + \frac{3}{2}\ell + \frac{1}{2}\ell + \frac{3}{4} - \ell^2 - \ell - \frac{3}{4}\right]$$

$$= \frac{\hbar^2}{2}[\ell] = \frac{\hbar^2 \ell}{2}$$

For $J = \ell - \frac{1}{2}$

$$\langle \ell \cdot s \rangle = \frac{\hbar^2}{2}\left[J(J+1) - \ell(\ell+1) - s(s+1)\right]$$

$$= \frac{\hbar^2}{2}\left[\left(\ell - \frac{1}{2}\right)\left(\ell + \frac{1}{2}\right) - \ell(+1) - \frac{3}{4}\right]$$

$$= \frac{\hbar^2}{2}\left[\ell^2 - \frac{1}{4} - \ell^2 - \ell - \frac{3}{4}\right] = -\frac{\hbar^2}{2}(\ell+1)$$

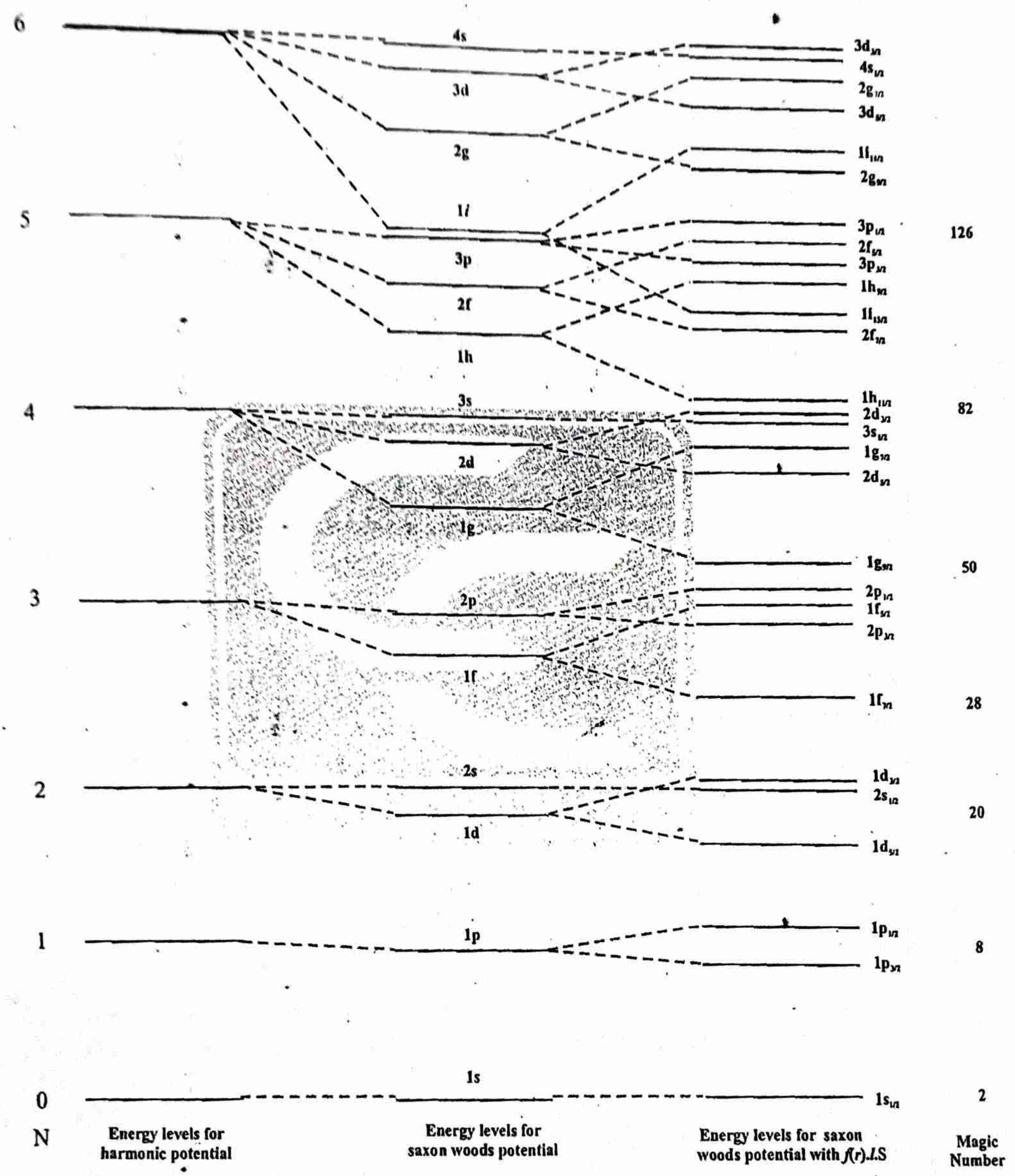
Since $f(r)$ have negative magnitude therefore $J = \ell + \frac{1}{2}$ have lower energy and $J = \ell - \frac{1}{2}$ have higher energy and energy splitting.

$$\Delta E = \frac{\hbar^2}{2}(2\ell+1)$$

i.e. higher l more energy gap.

We see that this larger spin-orbit effect leads to crossing over of energy levels into different shells. For example the state above the 2p state is 1g($l = 4$), which splits into $1g_{9/2}\left(J = \ell + \frac{1}{2}\right)$ and $1g_{7/2}\left(J = \ell - \frac{1}{2}\right)$.

The energy of the $1g_{9/2}$ state is sufficiently low that if joins the shell below. So that this forth shell now consists of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$ and $1g_{9/2}$. The maximum occupancy of this state ($(2J+1)$ proton or neutron for each J) is now 8 + 4 + 6 + 2 + 10 = 30 which added to the previous magic number 20, gives the next observed magic number of 50.



- **Success of shell model:**

1. It very well explains the existence of magic numbers and the stability and high binding energy on the basis of closed shells.
2. The shell model provides explanation for the ground state spins and magnetic moments of the nuclei. The neutrons and protons with opposite spins pair off so that the mechanical and magnetic moment cancel and the odd or left out proton or neutron contributes to the spin and magnetic moment of the nuclei as a whole.

3. Nuclear isomerism, i.e., existence of isobaric, isotopic nuclei in different energy states of odd-A nuclei between 39-49, 69-81, 111 to 125 has been explained by shell model by the large difference in nuclear spins of isomeric states are their A-values are close to magic numbers.

• Limitations of the shell model:

1. The model does not predict the correct value of spin quantum number J in certain nuclei, e.g., $^{23}_{11}Na$ where the predicted value is $J = 5/2$ while the correct value is $\frac{1}{2}$.
2. The following four stable nuclei ^2_1H; ^6_3Li, $^{10}_5B$ and $^{14}_7N$ do not fit into this model.
3. The model cannot explain the observed first excited states in even-even nuclei at energies much lower than those expected from single particle excitation. It also fails to explain the observed large quadrupole moment of odd-A nuclei, in particular of those having A-value far away from magic numbers.

Determination total angular momentum and parity of the ground state

1. If protons and neutrons both are even, then the total angular momentum become zero and parity is even.
2. If one of the nucleon (proton or neutron) is even and other is odd then the J value and $(-1)^l$ of the last unpaired neutron give the total angular momentum and parity.
3. For odd-odd nucleon we use Brennan-Bernstein rules. According to this rules

$$N = (J_p - \ell_p) + (J_n - \ell_n)$$

(i) If N = 0, then $J = |J_1 - J_2|$, $parity = -ve$

(ii) If N = ±1 and n = z then $J = |J_1 \pm J_2|$, $parity = \pm 1$

(iii) If N = ±1 and $n \neq z$ then $J = (J_p + J_n - 1)$, $parity = +ve$

Example :

$_8O^{16}$

$$z = 8 \text{ (even)} \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}$$

$$N = 8 \text{ (even)} \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}$$

Since, there have no unpaired nucleon.

Therefore, parity is positive and angular momentum J = 0. It is represented by O^+.

$^{27}_{13}Al$: $z = 13(\text{odd}) \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^5_{5/2}$

$N = 14(\text{even}) \rightarrow$ does not contribute in angular momentum

Therefor, J = 5/2; l = 5/2–1/2 = 2

Parity = $(-1)^2 = +1$, even parity. It is represented by $\frac{5^+}{2}$.

$^{33}_{16}S$: $z = 16(\text{even}) \rightarrow$ does not contribute in angular momentum

$N = 17(\text{odd}) \rightarrow 1s^2_{1/2}\, 1p^4_{3/2}\, 1p^2_{1/2}\, 1d^5_{5/2}\, 2s^2_{1/2}\, 1d^2_{3/2}$

Therefor, J = 3/2; l = 3/2+1/2 = 2

Parity = $(-1)^2 = +1$, even parity. It is represented by $\frac{3}{2}^+$.

$^{38}Cl_{17}$:

$z = 17(odd) \rightarrow 1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^6\, 2s_{1/2}^2\, 1d_{3/2}^1$ (particle state)

$n = 21(odd) \rightarrow 1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^6\, 2s_{1/2}^2\, 1d_{3/2}^4\, 1f_{7/2}^1$ (particle state)

$$N = (J_p - \ell_p) + (J_n - \ell n) = -\frac{1}{2} + \frac{1}{2} = 0$$

$$\therefore \quad J = |J_p - J_n| = \left|\frac{3}{2} - \frac{7}{2}\right| = \frac{4}{2} = 2 \text{ and parity} = -1$$

Therefore, spin parity = 2^-.

$^{26}Al_{13}$:

$z = 13(odd) = 1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^5$

$n = 13(odd) = 1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^5$

$$N = (J_p - \ell_p) + (J_n - \ell_n) = \frac{1}{2} + \frac{1}{2} = 1 \quad \text{and } n = z$$

$$J = |J_p + J_n| = \left|\frac{5}{2} + \frac{5}{2}\right| = 5, \text{ parity} = +ve \text{ and parity is } +ve$$

Therefore, spin parity 5^+.

$^{56}Co_{27}$:

$z = 27(odd) = 1s_{1/2}^2\, 1p_{3/2}^4\, 1p_{1/2}^2\, 1d_{5/2}^6\, 2s_{1/2}^2\, 1d_{3/2}^4\, 1f_{7/2}^7$ (hole state).

$n = 29(odd) = 1s_{1/2}^2\, 1p_{3/2}^2\, 1d_{5/2}^6\, 2s_{1/2}^2\, 1d_{3/2}^4\, 1f_{7/2}^8\, 2p_{3/2}^1$ (particle state)

$$N = (J_p - \ell_p) + (J_n - \ell_n) = \frac{1}{2} + \frac{1}{2} = 1 \quad \text{and } n \neq z$$

$$\therefore \quad J = |J_p + J_n - 1| = \frac{7}{2} + \frac{3}{2} - 1 = 4, \text{ parity} = +ve$$

Therefore, spin parity = 4^+.

Determination of magnetic moment $\langle \mu_z \rangle$:

Here, we assuming magnetic moment of whole nucleus is coming from unpaired last nucleon.

1. If last nucleon in $J = \ell + \frac{1}{2}$ then $\boxed{\langle \mu_z \rangle = \left[g_\ell \ell + g_s \left(\frac{1}{2}\right) \right] \mu_N}$

   (A) If the last nucleon is proton

   $g_\ell = 1,\ g_s = 5.5857$ and $\boxed{\langle \mu_z \rangle = [J + 2.2928]\, \mu_N}$

   (B) If the last nucleon is neutron

   $g_\ell = 0,\ g_s = -3.8260$ and $\boxed{\langle \mu_z \rangle = -1.9130\, \mu_N}$

2. If the last nucleon in $J = \ell - \frac{1}{2}$ then $\langle \mu_z \rangle = \frac{J}{2(J+1)}\left[g_\ell (2J+3) - g_s\right]\mu_N$

(A) If the last nucleon is a proton

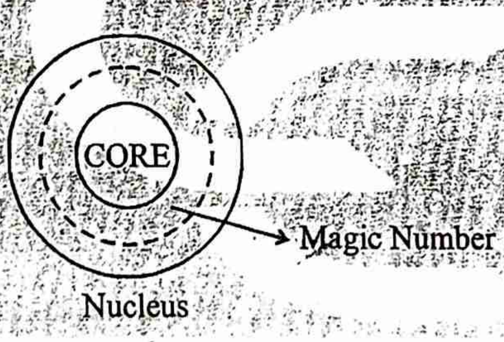
$$\langle \mu_z \rangle = \frac{J}{J+1}\left[J - 1.2928\right]\mu_N$$

(B) If the last nucleon is neutron

$$\langle \mu_z \rangle = \frac{J}{J+1} 1.9130 \mu_N$$

C. Collective Model:

- Liquid drop model fails to explain spin, magnetic moment, magic number.
- Shell model can not explain deviation of magnetic moment from schmidt limit and large quadrupole moment of certain nuclei and fission and fusion.
- A nucleus consist of a central CORE and some extra nucleons around the CORE. CORE consists of number of nucleons equal to the magic number.



The loose nucleons outside the CORE have large centrifugal force on the CORE. If their number is large, they are able to deform the CORE. The total energy of deformed nucleus can be written as:

$$E = E_N + E_V + E_R$$

E_N – due the extra nucleons
E_V – Vibrational motion
E_R – Because of rotational motion.
The rotational energy is

$$E_R \propto I(I+1)$$ Where I = spin of the rotational state.

Determination of quadruple moment of ground state:
If J is the spin of ground state then according to collective model the quadrupole is given by

$$Q = -\frac{3}{5}\left(\frac{2I-1}{2I+2}\right)R^2, \text{ where } R = 1.2\,A^{1/3}$$

D. Fermi gas model :

We assume the Fermi gas model in which the particles move without interaction in a sphere of radius R and volume V as the particles in an ideal gas do. According to the theory of the Fermi gas, the number of states of protons with momentum between momenta p and p + dp in a volume V is $\frac{2V\,4\pi p^2 dp}{(2\pi\hbar)^3}$. The spin-weight factor 2 originates from the two spin state of the proton. The ground state of the system will correspond to zero temperature, with momentum spread of zero to maxium P_{F_p} so that the number of protons in this lowest energy state is given by

$$z = \int_0^{P_{F_p}} \frac{2r4\pi p^2 dP}{(2\pi\hbar)^3} = \frac{r8\pi}{h^3}\frac{\left(P_{F_p}\right)^3}{3}$$

Therefore, the fermi momentum for the protons is

$$P_{F_p} = \hbar K_{F_p} = h\left(\frac{3z}{8\pi r}\right)^{1/3} = h\left(\frac{3}{8\pi}\rho_p\right)^{1/3}$$

where, $\partial_p = \frac{z}{v}$ is the proton density

Similarly, for the neutrons, we have

$$p_{F_n} = \hbar K_{F_N} = h\left(\frac{3N}{8\pi r}\right)^{1/3} = h\left(\frac{3}{8\pi}\rho_p\right)^{1/3}$$

where, $\rho_n = \frac{N}{r}$ is the neutron density. If we define the total nuclear density $\rho_A = \frac{A}{v} = \frac{(N+z)}{v}$ and assume that there are four particles in each state with equal numbers of neutrons and protons, the fermi momentum is given by

$$p_{F_A} = \hbar k_{F_A} = h\left(\frac{3A}{16\pi V}\right)^{1/3} = h\left(\frac{3}{16\pi}\rho_A\right)^{1/3}$$

The total kinetic energy of the protons in the ground state.

$$T_p = \int_0^{PF_p}\left(2r\frac{4\pi p^2 dp}{h^3}\frac{p^2}{2M}\right) = \frac{8\pi V}{2mh^3}\frac{\left(p_{F_p}\right)^5}{5} = \frac{4\pi}{5M}\left(\frac{3}{8\pi}\right)^{5/3}\frac{h^2}{r^{2/3}}z^{5/3}$$

Similarly, the total kinetic energy for neutrons is

$$T_N = \frac{4\pi}{5M}\left(\frac{3}{8\pi}\right)^{5/3}\frac{h^2}{r^{2/3}}N^{5/3}$$

Therefore, the total kinetic energy of N neutrons and z protons is then,

$$\boxed{K.E. = \frac{4\pi}{5M}\left(\frac{3}{8\pi}\right)^{5/3}\frac{h^2}{r^{2/3}}\left(z^{5/3} + N^{5/3}\right)}$$

SOLVED PROBLEMS

1. The angular momentum and parity of '$^{17}_{8}O$' nucleus, according to it, the nuclear shell model (including spin-orbit coupling) is: [GATE 1998]

(a) 0^+ (b) $1/2^-$ (c) $3/2+$ (d) $5/2^+$

Soln. We have nucleus, $^{17}_{8}O$

Therefore, proton number, z = 8 is even and neutrons number N = 9 is odd.

$$9N \rightarrow 1s_{1/2}^2\ 1p_{3/2}^4\ 1p_{1/2}^2\ 1d_{5/2}^1$$

$\therefore \quad J=\frac{5}{2},\ \ell=2,\ \text{parity}=(-1)^2=+\text{ve}$

Therefore, spin parity of ground state is $\frac{5}{2}^+$.

Correct option is (d)

Common Data for Q.2 and Q.3:

The nucleus ^{41}Ca can be described by the single particle shell model.

2. The single particle states occupied by the last proton and the last neutron, respectively, are given by **[GATE 2004]**

(a) $d_{5/2}$ and $f_{7/2}$ (b) $d_{3/2}$ and $f_{5/2}$ (c) $d_{5/2}$ and $f_{5/2}$ (d) $d_{3/2}$ and $f_{7/2}$

Soln. We have nucleus, $^{41}Ca_{20}$

$z=20\ (\text{even}) \rightarrow 1s_{1/2}^2\ 1p_{3/2}^4\ 1p_{1/2}^2\ 1d_{5/2}^6\ 2s_{1/2}^2\ 1d_{3/2}^4$

$N=21\ (\text{odd}) \rightarrow 1s_{1/2}^2\ 1p_{3/2}^4\ 1p_{1/2}^2\ 1d_{5/2}^6\ 2s_{1/2}^2\ 1d_{3/2}^4\ 1f_{7/2}^1$

Therefore, the last proton occupied $d_{3/2}$ state and last neutron occupied $f_{7/2}$ state.

Correct option is (d)

3. The ground state angular momentum and parity of ^{41}Ca are: **[GATE 2004]**

(a) $\left(\frac{7}{2}\right)^-$ (b) $\frac{3^+}{2}$ (c) $\frac{5^+}{2}$ (d) $\frac{5^-}{2}$

Soln. We have nucleus, $^{41}Ca_{20}$

$z=20\ (\text{even}) \rightarrow$ doesn't contribute in angular momentum.

$N=21\ (\text{odd}) \rightarrow 1s_{1/2}^2\ 1p_{3/2}^4\ 1p_{1/2}^2\ 1d_{5/2}^6\ 2s_{1/2}^2\ 1d_{3/2}^4\ 1f_{7/2}^1$

$J=\frac{7}{2},\ \ell=3\ \text{parity}=(-1)^3=-\text{ve}$

Spin parity of ground state $\frac{7}{2}^-$

Correct option is (a)

4. According to the shell model, the ground state of $^{15}_{8}$O nucleus is: **[GATE 2005]**

(a) $\frac{3^+}{2}$ (b) $\frac{1^+}{2}$ (c) $\frac{3^-}{2}$ (d) $\frac{1^-}{2}$

Soln. We have nucleus $^{15}_{8}O$

$z=8\ (\text{even})$

$N=7\ (\text{odd}) \rightarrow 1s_{1/2}^2\ 1p_{3/2}^4\ 1p_{4/2}^1$

$\therefore \quad J=\frac{1}{2},\ \ell=1\ \text{partity}=(-1)^1=-\text{ve}$

$\therefore \quad$ Ground state spin parity $=\frac{1}{2}^-$

5. The four possible configurations of neutrons in the ground state of $_4Be^9$ nucleus, according to the shell model, and the associated nuclear spin are listed below. Choose the correct one:

(a) $(1s_{1/2})^2\,(1P_{3/2})^3;\ J=\frac{3}{2}$ (b) $1(s_{1/2})^2\,(1P_{1/2})^2\,(1p_{3/2})^1;\ J=\frac{3}{2}$ **[GATE 2008]**

(c) $(1s_{1/2})^2\,(1P_{3/2})^4;\ J=\frac{1}{2}$ (d) $1(s_{1/2})^2\,(1P_{3/2})^2\,(1p_{1/2})^1;\ J=\frac{1}{2}$

Soln. We have nucleus $_4Be^9$

$Z=4$ (even)

$N=5$ (odd) $\rightarrow 1s_{1/2}^2\,1p_{3/2}^3$

4]

$\therefore\ J=\frac{3}{2}$

Correct option is (a)

6. In the nuclear shell model, the potential is modeled as $V(r)=\frac{1}{2}m\omega^2r^2-\lambda\vec{L}\cdot\vec{S},\ \lambda>0$. The correct spin-parity and isospin assignments for the ground state of ^{13}C is **[GATE 2015]**

(a) $\frac{1^-}{2};\frac{-1}{2}$ (b) $\frac{1^+}{2};\frac{-1}{2}$ (c) $\frac{3^+}{2};\frac{1}{2}$ (d) $\frac{3^-}{2};\frac{-1}{2}$

004]

Soln. We have nucleus, $^{13}C_6$

$Z=6$ (even)

$N=7$(odd) $\rightarrow 1s_{1/2}^2\,1p_{3/2}^4\,1p_{1/2}^1$

$J=\frac{1}{2},\ \ell=1,\ \text{parity}=(-1)^1=-1$

Spin parity of ground state is $\frac{1^-}{2}$

Isospin, $I=\frac{M-1}{2}=\frac{2-1}{2}=\frac{1}{2}$ (M is the multiplicity $^{13}C,\,^{12}C$)

2005] Therefore, Isospin I_3 of ^{13}C is $-\frac{1}{2}$

Correct option is (a)

The following gives a list of pairs containing (i) a nucleus (ii) one of its properties. Find the pair which is inappropriate: **[GATE 2008]**

(a) (i) $_{10}Ne^{20}$ nucleus; (ii) stable nucleus

(b) (i) A spheroidal nucleus; (ii) an electric qudrupole moment

(c) (i) $_8O^{16}$ nucleus; (ii) nuclear spin $J=\frac{1}{2}$

(d) U^{238} nucleus; (ii) Binding energy = 1785 MeV (approximately)

oln. (a) $_{10}Ne^{20}$:

$Z=10$; $N=10$

Therefore, N/Z = 1

Therefore, nucleus is stable.

(b) Spherical shell, Q = 0

Spheroidal nucleus Q > 0 or Q < 0

(c) $_8O^{16}$:

Z = 8; N = 8

Since both nucleons are even. So, spin parity of the ground state is 0^+.

(d) Since, each nucleons of U^{238} has binding energy 7.5 MeV.

So, total binding energy of U^{238} will be $7.5 \times 239 \simeq 1785\ MeV$.

Correct option is (c)

8. If the masses of ${}^{40}_{20}Ca$, ${}^{41}_{20}Ca$ and ${}^{39}_{20}Ca$ are 39.962589 a.m.u, 40962275 a.m.u and 38.970691 a.m.u respectively, calculate the energy difference between $1d_{3/2}$ and $1f_{7/2}$ neutron shells.

Soln. Ca-39 has one neutron missing in the $1d_{3/2}$ shell; Ca-40 completes this shell and Ca-41 adds a neutron to $1f_{7/2}$ shell.

Therefore, B.E. of $1d_{3/2}$ neutron in Ca-40 is:

$$B_1 = (M_{39} + M_n - M_{40})c^2 = (38.97069 + 1.008665 - 39.962589) \times 931\ \text{MeV}$$
$$= 0.016766 \times 931\ \text{MeV} = 15.61\ \text{MeV}$$

B.E. of $1f_{7/2}$ neutron in Ca-40 is:

$$B_2 = (M_{40} + M_n - M_{41})c^2$$
$$= (39.962589 + 1.008665 - 40.96225) \times 931\ \text{MeV}$$
$$= 0.008979 \times 931\ \text{MeV} = 8.36\ \text{MeV}$$

Therefore, the energy separation ΔE is the difference $B_1 - B_2$

$\therefore \quad \Delta E = B_1 - B_2 = 15.61 - 8.36 = 7.25\ \text{MeV}$

9. Find the total angular momentum and parity for the ground state of ${}^{33}_{16}S$ nucleus, using the shell model, and also its electric quadrupole moment from the collective model.

Soln. ${}^{33}_{16}S$:

$$16P - \left(1s_{\frac{1}{2}}\right)^2 \left(1p_{\frac{3}{2}}\right)^2 \left(1p_{\frac{1}{2}}\right)^2 \left(1d_{\frac{5}{2}}\right)^6 \left(2s_{\frac{1}{2}}\right)^2$$

$$17N - \left(1s_{\frac{1}{2}}\right)^2 \left(1p_{\frac{3}{2}}\right)^4 \left(1p_{\frac{1}{2}}\right)^2 \left(1d_{\frac{5}{2}}\right)^2 \left(2s_{\frac{1}{2}}\right)^2 \left(1d_{\frac{3}{2}}\right)^1$$

The total angular momentum or spin of the nucleus ${}^{33}_{16}S$ is the total angular momentum of the last unpaired neutron.

$\therefore \quad J = 3/2; \quad = 2$ for d state

$\therefore \quad \text{Parity} = (-1)^2 = +1$, even parity

The electric quadrupole moment, Q of a nucleus with spin J is given, according to collective model, by

$$Q=-\frac{3}{5}\left(\frac{2J-1}{2J+2}\right)R_0^2$$

where $R_0=1.2\times A^{1/3}\ \text{fm}=1.2\times(33)^{1/3}\times10^{-15}\text{m}$ (since A=33 here)

$$\therefore \quad Q=-\frac{3}{5}\left\{\frac{\left(2\times\frac{3}{2}\right)-1}{2\times\frac{3}{2}+2}\right\}\times\left[1.2\times(33)^{1/3}\times10^{-15}\right]^2$$

$$=-0.0355\times10^{-28}\ \text{m}^2=-0.0355\ \text{barn} \qquad \left(\because 1\ \text{barn}=10^{-28}\ \text{m}^2\right)$$

10. Compute the binding energy of the last proton in a nucleus of ^{12}C if the mass of ^{12}C-nucleus is 12.00052 was a.m.u. and the mass of the ^{11}B-nucleus is 11.01006 a.m.u. The mass of proton is 1.00759 a.m.u

Soln. On the addition of a proton, the ^{11}B-nucleus is converted into ^{12}C nucleus. The excess of mass of ^{12}C over ^{11}B is:

$$12.00052-11.01006=0.99046\ \text{a.m.u.}$$

The mass of proton is 1.00759 a.m.u. Thus the proton, when added to the nucleus, suffers mass loss.

Mass loss, $\Delta m=1.00759-0.99046=0.01713$ a.m.u.

$\therefore$ Equivalent energy, $\Delta E=0.01713\times931=15.95$ MeV

$\therefore$ Binding energy of the last proton = 15.95 MeV

11. Establish the relation $A=2Z$ for light nuclei using the semi-empirical mass formula, given $a_c=0.71\ \text{MeV}, a_n=22.7\ \text{MeV}, M(^1_1H)=1.0078, M(n)=1.0086$ unit.

Soln. The mass M of a nucleus of mass number A and charge number Z according to the semi-empirical formula, is given by

$$M=Z M_H+(A-Z)M_n-\frac{1}{c^2}\left(a_vA-a_sA^{2/3}-a_c\frac{Z^2}{A^{1/3}}-\frac{a_n(A-2Z)^2}{A}\pm E_\delta\right)$$

For odd A nuclei, we have $E_\delta=0$

The mass of the most stable nucleus in a family of isobars is given by the condition $(\partial M/\partial Z)_A=0$.

$$\Rightarrow \quad (\partial M/\partial Z)_A=(M_H-M_n)c^2+2a_c\frac{Z}{A^{1/3}}-4a_n\frac{(A-2Z)}{A}=0$$

$$\Rightarrow \quad 2a_c\frac{Z}{A^{1/3}}-4a_n\frac{(A-2Z)}{A}=(M_n-M_H)c^2$$

$$\Rightarrow \quad 2Z\left(\frac{a_c}{A^{1/3}}+\frac{4a_n}{A}\right)=(M_n-M_H)c^2+4a_n$$

$$\Rightarrow \quad \frac{2Z}{A}\left(a_cA^{2/3}+4a_n\right)=(M_n-M_H)c^2+4a_n$$

$$\therefore \quad \frac{2Z}{A}\left\{\frac{a_cA^{2/3}}{4a_n}+1\right\}=\left\{\frac{(M_n-M_H)c^2}{4a_n}+1\right\}$$

Or, $Z = \frac{A}{2}\left\{\frac{1+(M_n - M_H)c^2/4a_n}{1+\frac{a_c}{4a_n}A^{2/3}}\right\}$

Now, $a_c = 0.71\,\text{MeV},\ a_n = 22.7\,\text{MeV}$. $\quad \therefore\ a_c/4a_n = 0.0078$ and $(M_n - M_H)c^2/4a_n = 0.0082$

$$\therefore \quad Z = \frac{A}{2}/\left\{\frac{1+0.0082}{1+0.0078\,A^{2/3}}\right\} \simeq \frac{A}{2}, \text{ for light nuclei.}$$

$\therefore\ A \simeq 2Z$, for light nuclei.

12. Using the semi-empirical binding energy formula, calculate the binding energy of $^{40}_{20}Ca$.

Soln. The semi-empirical binding energy formula is:

$$B.E. = a_v A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} + \delta A^{-3/4}$$

where $a_v = 15.75\,\text{MeV}$, $a_s = 17.80\,\text{MeV}$, $a_c = 0.71\,\text{MeV}$,

$a_n = 22.7\,\text{MeV}$ and $\delta = 34$ as $A = \text{even} = 40$, $Z = \text{even} = 20$

$\therefore \quad a_v A = 15.75 \times 40 = 630\,\text{MeV}$; $\ a_s A^{2/3} = 17.80 \times 40^{2/3} = 17.80 \times 11.696 = 208.2\,\text{MeV}$

$$a_c \frac{Z(Z-1)}{A^{1/3}} = \frac{0.71 \times 20 \times 19}{40^{1/3}} = \frac{0.71 \times 20 \times 19}{3.2} = 84.3\,\text{MeV}$$

$$a_n \frac{(A-2Z)^2}{A} = a_n \times 0 = 0$$

$$\delta A^{-3/4} = 34 \times 40^{-3/4} = 34 \times 0.063 = 2.14\,\text{MeV}$$

$\therefore \quad B.E. = 630 - [208.2 + 84.3 - 2.14] = 339.64\,\text{MeV}$

13. Using the semi-empirical binding energy formula, find the atomic number of the most stable nucleus for a given mass number A. Hence explain which is the most stable among ^5_2He, ^6_2Be and ^6_3Li

Soln. Writing E_b for binding energy, $E_b = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_n \frac{(A-2Z)^2}{A} \pm \frac{\delta}{A^{3/4}}$

where, $Z(Z-1) \simeq Z^2$ has been taken.

Now, for most stable nucleus, E_b must be maximum for a given mass number A, i.e.,

$$\left(\frac{\partial E_b}{\partial Z}\right)_{A=\text{const}} = -2a_c A^{-1/3} Z + 4a_n (A-2Z)A^{-1} = 0$$

$$\Rightarrow \quad 4a_n - 8a_n A^{-1} Z = 2a_c A^{-1/3} Z;\quad Z\left(4a_n + a_c A^{2/3}\right) = 2a_n A$$

$$\therefore \quad Z = \frac{A}{2 + (a_c/2a_n)A^{2/3}} = \frac{A}{2 + 0.015\,A^{2/3}}$$

Substituting the values of a_c and a_a.

He, Be and Li are all light nuclei for which $0.015\ A^{2/3}$ is negligible and $Z = A/2$. This shows that of the three nuclei, ${}^{6}_{3}\text{Li}$ is most stable.

14. Show, by way of computation, which nuclei you would expect to be more stable:

$$ {}^{7}_{3}\text{Li} \text{ or } {}^{8}_{3}\text{Li};\ {}^{9}_{4}\text{Be} \text{ or } {}^{10}_{4}\text{Be} $$

Soln. For a given mass number A, the atomic number Z of the most stable nucleus is:

$$ Z = \frac{A}{2 + 0.015\,A^{2/3}} $$

Now, for $A = 7,\ Z = \frac{7}{2 + 0.015 \times 7^{2/3}} = \frac{7}{2 + 0.055} = \frac{7}{2.055} = 3.4$

for $A = 8,\ Z = \frac{8}{2 + 0.15 \times 8^{2/3}} = \frac{8}{2 + 0.060} = \frac{8}{2.060} = 3.88$

Since of the two Z-values, 3.4 is nearer to 3, the ${}^{7}_{3}\text{Li}$ nucleus is more stable.

Again, for $A = 9,\ Z = \frac{9}{2 + 0.015 \times 9^{2/3}} = \frac{9}{2 + 0.065} = \frac{9}{2.065} = 4.36$

for $A = 10,\ Z = \frac{10}{2 + 0.015 \times 10^{2/3}} = \frac{10}{2 + 0.067} = \frac{10}{2.067} = 4.80$

Since the two Z-values, 4.36 is nearer to 4, the ${}^{9}_{4}\text{Be}$ nucleus is more stable.

5. Consider a nuclear level corresponding to a closed shell plus a single proton in a state with the angular momentum quantum numbers l and j. Of course $j = \ell \pm 1/2$. Let g_p be the empirical gyromagnetic ratio of the free proton. Compute the gyromagnetic ratio for the level in question, for each of the two cases $j = \ell + 1/2$ and $j = \ell - 1/2$.

oln. According to the shell model, the total angular momentum of the nucleons in a closed shell is zero, so is the magnetic moment. This means that the magnetic moment and angular momentum of the nucleus are determined by the only proton outside the closed shell.

As, $\mu_j = \mu_\ell + \mu_s,\ g_j j = g_\ell \ell + g_s S$

We have, $g_j j.j = g_\ell \ell.j + g_s s.j$

With
$$ \ell.j = \frac{1}{2}\left(j^2 + \ell^2 - s^2\right) = \frac{1}{2}\left[j(j+1) + \ell(\ell+1) - s(s+1)\right] $$

$$ s.j = \frac{1}{2}\left(j^2 + s^2 - \ell^2\right) = \frac{1}{2}\left[j(j+1) + s(s+1) - \ell(\ell+1)\right] $$

$$ g_j = g_\ell \frac{j(j+1) + \ell(\ell+1) - s(s+1)}{2j(j+1)} + g_s \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)} $$

For proton, $g_\ell = 1,\ g_s = g_p$ the gyromagnetic ratio for free proton $(\ell = 0, j = s), s = \frac{1}{2}$. Hence we have

$$ g_j = \begin{cases} \dfrac{2j-1}{2j} + \dfrac{g_p}{2j} & \text{for } j = \ell + 1/2 \\[2ex] \dfrac{1}{j+1}\left(j + \dfrac{3}{2} - \dfrac{g_p}{2}\right) & \text{for } j = \ell - 1/2 \end{cases} $$

The simplest model for low-lying states of nuclei with N and Z between 20 and 28 involves only $f_{7/2}$ nucleons.

16. Using this model in previous question, predict the magnetic dipole moments of $^{41}_{20}Ca$ and $^{41}_{21}Sc$.

Soln. ^{41}Ca has a neutron and ^{41}Sc has a proton outside closed shells in state $1f_{7/2}$. As closed shells do not contribute to the nuclear magnetic moment, the latter is determined by the extra-shell nucleons. The nuclear magnetic moment is given by $\mu = g_j \mu_n$

where 'j' is the total angular momentum, μ_n is the nuclear magneton. for a single nucleon in a central field the g-factor is

$$g = \frac{(2j-1)g_\ell + g_s}{2j} \quad \text{for } j = \ell + \frac{1}{2}$$

$$g = \frac{(2j+1)g_\ell - g_s}{2(j+1)} \quad \text{for } j = \ell - \frac{1}{2}$$

For neutron, $g_\ell = 0, g_s = g_n = -\frac{1.91}{\frac{1}{2}} = -3.82$. As $\ell = 3$ and $j = \frac{7}{2} = 3 + \frac{1}{2}$, we have for ^{41}Ca.

$$\mu\left(^{41}_{20}Ca\right) = -\frac{3.82}{2j} \times j\mu_N = -1.91\,\mu_N$$

For proton, $g_\ell = 1, g_s = g_p = \frac{2.79}{1/2} = 5.58$. As $j = \frac{7}{2} = 3 + \frac{1}{2}$, we have for ^{41}Sc.

$$\mu\left(^{41}_{21}Sc\right) = \frac{(7-1)+5.58}{7} \times \frac{7}{2}\mu_N = 5.79\,\mu_N$$

17. The band spectrum of U^{238} base on $(O)^+$ ground state. If the energy of $(2)^+$ state is 44.7 KeV. What is the spin and parity of state having energy 5.25 KeV.

Soln. Since $E_I \propto I(I+1)$

$$E_I = k\,I(I+1) \quad \text{... (1)}$$

$$E_2 = k\,2(2+1) \quad \Rightarrow \quad E_2 = k\,2\times 3 \quad \text{... (2)}$$

equation (2) divided by (1)

$$\Rightarrow \quad \frac{E_2}{E} = \frac{6}{I(I+1)} \quad \Rightarrow \quad \frac{6}{I(I+1)} = \frac{44.7}{525}$$

$$\Rightarrow \quad I(I+1) = \frac{6\times 525}{44.7} \approx 72 \quad \Rightarrow I^2 + I - 72 = 0 \quad \Rightarrow I^2 + 9I - 8I - 72 = 0$$

$$\Rightarrow \quad I(I+9) - 8(I+9) = 0 \quad \Rightarrow (I-8)(I+9) = 0$$

$I = +8 \qquad I \neq -\text{ive}$

Or, $\boxed{I = 8^+} \qquad I^P = 8^+$

18. The rotation energy level of an even-even nucleus are (0^+) ground state, energy of $2^+ = 44$ KeV and higher state energies are 146, 304, 514 KeV respectively for three exited state. Assign spin and parity to these levels.

Soln. (i) $E_I \propto I(I+1) \Rightarrow E_I = KI(I+1)$

$E_2 = k\,2\times 3$

$E = k\,I(I+1)$

$$\frac{E_2}{E} = \frac{6}{I(I+1)} \Rightarrow \frac{6}{I(I+1)} = \frac{44}{146} \Rightarrow I(I+1) = \frac{6\times 146}{44} = 6\times 3.3 = 19.8 = 20$$

$\Rightarrow I^2 + I = 20 \Rightarrow I^2 + I - 20 = 0 \Rightarrow I^2 + 5I - 4I - 20 = 0$

$\Rightarrow I(I+5) - 4(I+5) = 0$

$\Rightarrow (I-4) = 0 \Rightarrow \boxed{I=4}$

(ii) $$\frac{E_2}{E_3} = \frac{I(I+1)}{I(I+1)} \Rightarrow \frac{146}{304} = \frac{4(4+1)}{I(I+1)} \Rightarrow I(I+1) = \frac{20\times 304}{146} = \frac{3040}{73}$$

$\Rightarrow I^2 + I = 41.64 \approx 42 \Rightarrow I^2 + I - 42 = 0 \Rightarrow I^2 + 7I - 6I - 42 = 0$

$\Rightarrow I(I+7) - 6(I+7) = 0 \Rightarrow (I+7)(7-6) = 0 \Rightarrow \boxed{I=6}$

(iii) $$\frac{E_3}{E_4} = \frac{I(I+1)}{I(7+1)} \Rightarrow \frac{304}{514} = \frac{6\times 7}{I(I+1)}; \quad I^P = 8^+$$

19. The difference in the coulomb energy between the mirror nuclei $^{49}_{24}Cr$ and $^{49}_{25}Mn$ is 6MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that e^2 is approximately 1.0 MeV-fm. Find the radius of the $^{49}_{25}Mn$ nucleus.

Soln. Kinetic coulomb energy of uniformly charged sphere of radius R is

$$E_C = \frac{3}{5}\frac{z^2e^2}{R}$$

According to the question, $E_{Cr} - E_{Mn} = 6MeV$

$$\frac{3}{5}\frac{(Z_{Cr}^2 - Z_{Mn}^2)e^2}{R} = 6MeV \Rightarrow R = \frac{3(Z_{Cr}^2 - Z_{Mn}^2)e^2}{6\times 5MeV}$$

$= 4.9$ fm $= 4.9 \times 10^{-15}$ fm

where $Z_{Cr} = 25$ and $Z_{Mn} = 24$

20. For what elements should stable isobars exist for (a) A = 97 and (b) A = 80?

Soln. Semi-empirical mass formula for a $_zX^A$-atom is

$$_zM^A = M_N(A-Z) + M_HZ - a_vA + a_sA^{2/3} + a_cZ(Z-1)A^{-1/3} + a_a(A-2Z)^2A^{-1} \pm a_pA^{-3/4}$$

Substituting values of constants in terms of MeV, we have

$${}_Z M^A = 939.6(A-Z) + 938.8Z - 14A + 13A^{2/3} + 0.60Z(Z-1)A^{-1/3}$$

$$+19(A-2Z)^2 A^{-1} \pm 34A^{-3/4}$$

For a most stable isobar

$$(\partial M / \partial N)_{z-zA} = -0.8 + 0.60(2Z_A - 1)A^{-1/3} - 76(A-2Z_A)A^{-1} = 0$$

$$\text{or } Z_A = \frac{76.8 + 0.6A^{-1/3}}{152 + 1.2A^{2/3}} A$$

For odd value of A, there should be only one stable isobar, the neighbouring isobars are unstable and decay to stable isobar. for A = 97.

$$Z_A = \frac{76.8 + 0.6 \times (97)^{-1/3}}{152 + 1.2(97)^{2/3}} (97) = 42.1$$

Hence the most stable isobar is ${}_{42}Mo^{97}$.

By similar calculations for A = 80, we get Z_A = 35.32. Hence the most stable isobar will be ${}_{35}Br^{80}$. It is the case of Z-odd, N-odd. It has been discussed in theory that for even-A values, we get two parabolas due to the term $\pm a_V A^{-3/4}$. The odd Z values are associated with upper and even Z-values with the lower. The possible stable isobars with Z-even values are ${}_{34}Se^{80}$ and ${}_{36}Kr^{80}$.

PRACTICE SET

1. According to the shell model of the nucleus
(a) magic numbers exist.
(b) nucleons interact with their nearest neighbours only.
(c) nucleons in a nucleus interact with a general force field.
(d) large electron quadrupole moment exists for certain nuclei.

2. The nuclear spins of ${}_6C^{14}$ and ${}_{12}Mg^{25}$ nuclei and respectively:
(a) zero and half-integer (b) half-integer and zero
(c) an integer and half-integer (d) both half-integers

3. The spin and parity of ${}_4Be^9$ nucleus, as predicted by the shell model, are respectively:
(a) 3/2 and odd (b) 1/2 and odd (c) 3/2 and even (d) 1/2 and even

4. The expression of a_c is:

(a) $\frac{3e^2}{4\pi\epsilon_0 R_0}$ (b) $\frac{3e^2}{10\pi\epsilon_0 R_0}$ (c) $\frac{3e^2}{20\pi\epsilon_0 R_0}$ (d) $\frac{e^2}{20\pi\epsilon_0 R_0}$

5. The binding energy of a light nucleus (Z, A) in MeV is given by the approximate formula

$$B(A,Z) \approx 16A - 20A^{2/3} - \frac{3}{4}Z^2A^{-1/3} + 30\frac{(N-Z)^2}{A}$$

where N = A – Z is the neutron number. The value of Z of the most stable isobar for a given A is

(a) $\frac{A}{2}\left(1-\frac{A^{2/3}}{160}\right)^{-1}$ (b) $\frac{A}{2}$ (c) $\frac{A}{2}\left(1-\frac{A^{2/3}}{120}\right)^{-1}$ (d) $\frac{A}{2}\left(1+\frac{A^{4/3}}{64}\right)$

ANSWER KEY					
Questions	1	2	3	4	5
Option	(a)	(a)	(a)	(b)	(b)

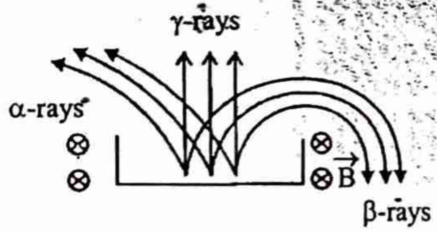
Radioactivity

Introduction:

In 1903, the Curies were awarded jointly with Becquerel, the Nobel Prize in physics for his discovery of radioactivity phenomenon. In this chapter, I would like to discuss radioactive decay law, α decay, β decay, γ decay.

Radioactivity:

Radioactivity is the phenomenon of spontaneous distintegration, attended with emission of corpuscular or electromagnetic radiations, of heavy atomic nuclei $\left(\text{for } \frac{n}{z} > 1.5\right)$ like uranium radium etc at a contant rate unaffected by any physical or chemical changes or influences such as temperature, pressure etc. to which the atom may be subjected. It is a nuclear property of the active element and in all radioactive processes a transmutation of the element occure an altogether new nucleus is formed.



Radioactive Decay law:

1. On emission of α or β rays, which is usually but not invariably accompanied by γ-ray emission, the emitting parent nuclei transforms into a new daughter element. The daughter element again is radioactive so that the process of successive disintegration continuous till the original active parent nuclei gets transformed into stable one.

2. The rate of radioactive disintegration, that is, the number of atoms that break up at any instant of time 't' is directly proportional to the number N_t of active nucleids present in the sample under study at that instant.

Decay equation:

Let N_t be the number of active nucleides present in the sample at any time 't' then we have experimentally

$$-\frac{dN_t}{dt} \propto N_t$$

$$\Rightarrow \quad \frac{dN_t}{dt} = -\lambda N_t$$

Where, λ, the constant of proportionality, is known as the *decay* constant – a characteristic constant of the element (nuclide). The negative sign hints at the fact that N_t decreases with t.

$$\frac{dN_t}{N_t} = -\lambda dt$$

$$\ell n\, N_t = -\lambda t + A$$

where, A is the constant of integration.

At $t = 0$, $N_t = N_0$, the initial number of nuclides and $A = \ell n\, N_0$

Therefore, $$\ell n\left(\frac{N_t}{N_0}\right) = -\lambda t$$

Or, $$\boxed{N_t = N_0 e^{-\lambda t}}$$

Half-life: The half-life of a radioactive nuclide is defined as the time $T_{1/2}$ in which the original amount of of radioactive atoms is reduced by way of distintegration to half its value.

At $t = T_{1/2}$, $N_t = N_0/2$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\therefore \quad T_{1/2} = \frac{\ell n 2}{\lambda} = \frac{2.303 \log 2}{\lambda} = \frac{0.693}{\lambda}$$

Or, $$\boxed{\lambda T_{1/2} = 0.693 = \text{constant}}$$

Average life:

The average or mean life time $\bar{T}$ of a radioelement is the average life time of all the atoms in the given sample and is defined as the ratio of the total life time of all the atoms to be total number of atoms.

$$\therefore \quad \bar{T} = \frac{t_1 dN_1 + t_2 dN_2 + \ldots..}{dN_1 + dN_2 + \ldots...} = \frac{\Sigma t dN}{\Sigma dN} = \frac{\int_{t=0}^{\infty} t dN}{\int_{N_0}^{0} dN} = \frac{\int_{t=0}^{\infty} t dN}{-N_0}$$

But, we have,

$$dN = d\left(N_0 e^{-\lambda t}\right) = -\lambda N_0 e^{-\lambda t} dt$$

$$\therefore \quad \bar{T} = \lambda \int_0^{\infty} t e^{-\lambda t} dt = \lambda\left[-\frac{t}{\lambda} e^{-\lambda t} + \frac{1}{\lambda}\int_0^{\infty} e^{-\lambda t} dt\right]$$

$$= \lambda\left[-\frac{t}{\lambda} e^{-\lambda t} - \frac{1}{\lambda^2} e^{-\lambda t}\right]_0^{\infty} = \frac{1}{\lambda}$$

Therefore, $$\boxed{\bar{T} = \frac{1}{\lambda}}$$

We can write also, $$\boxed{T_{1/2} = \frac{0.693}{\lambda} = 0.693\bar{T}}$$

Activity:

The activity or strength A_t of a radioactive sample at any instant t is thus defined as the number of disintegration occurring in the sample in unit time at t, that is,

$$\text{Activity,}\quad A_t = \left|\frac{dN_t}{dt}\right| = \lambda N_t = \frac{0.693}{T} N_t$$

Units of activity:

The customer unit of radioactivity is called the curie (Ci). It is defined as the acitivyt of any radioactive substance that disintegrates at the rate of 3.7×10^{10} disintegration per second.

$$1\,Ci = 1\,curie = 3.7\times10^{10}\ \text{disint/sec}$$

$$1\,mCi = 10^{-3}\ curie$$

$$1\,\mu Ci = 10^{-6}\ curie$$

Radioactive displacement law:

1. The α particles having been identities as helium nuclei. Suppose I have parent nucleus having mass number A and atomic number, after α decay the parent nucleus become a new daughter element whose atomic number $z-2$ and mass $A-4$. It position in periodic table will be shifted towards left by two.

$$^{A}X_{z} \xrightarrow{\alpha} {}^{A-4}Y_{z-2}$$

2. The β-particle defined as $_{-1}e^{0}$ due to β emission the parent elements $(z-A)$ become a daughter element $(z+1, A)$. So, it position in periodic table will be shifted towards right by one.

$$^{A}X_{z} \xrightarrow{\beta} {}^{A}Y_{z+1}$$

Natural radioactive decay series:

There are three naturally occuring radioactive series. These three series are

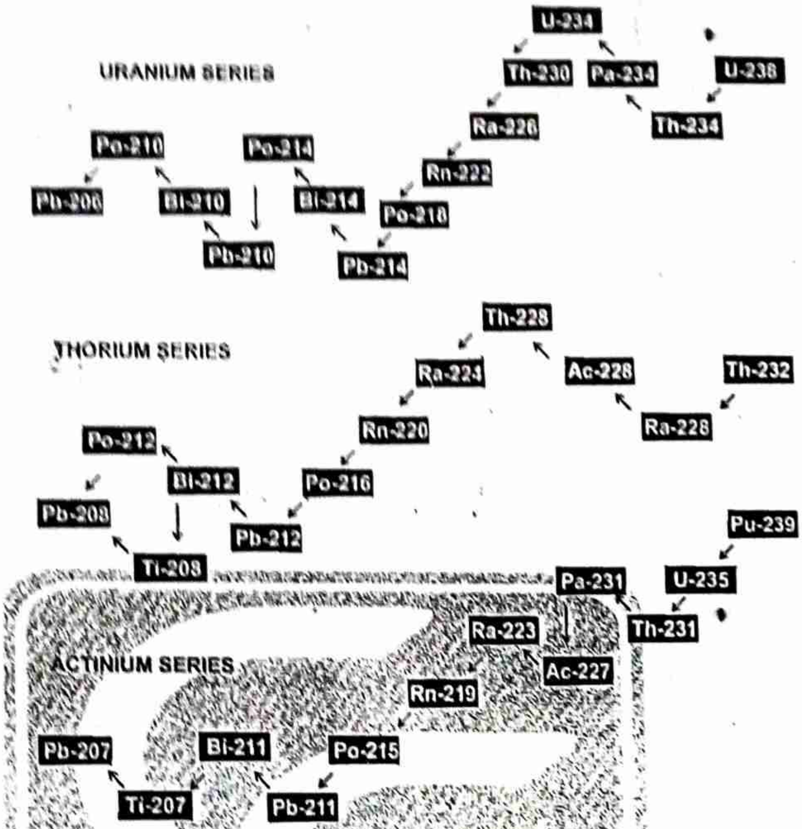
(i) uranium series is called (4n + 2)

(ii) actinium series is called (4n + 3)

(iii) therium series is called (4n)

Also there have another series is called Neptunium series

(iv) Neptunium series is called (4n + 1)



Note: ↖ $-(\beta\text{-decay})$, ↙ $-(\alpha\text{-decay})$

Successive transformation:

Consider a radioactive nuclide *P, symbolised by script 1, to decay into another radioactive nuclide Q (script 2); the latter again decays into a stable end-product R (script 3). For instance,

$$P(1) \xrightarrow{\lambda_1} Q(2) \xrightarrow{\lambda_2} R(3)\ [stable]$$

Let λ_1, λ_2 be the decay constants of nuclides 1 and 2 respectively, and N_1, N_2, N_3 be the number of atoms of the three kinds at any intant t.

Now the second nuclide would be formed at the rate $\lambda_1 N_1$ by the decay of the parent atom and disappear by its own decay at the rate $\lambda_2 N_2$; the atoms of the end-product i.e. R appear at the rate $\lambda_2 N_2$ by the decay of nuclide 2, but being stable do not disappear. So,

$$\frac{dN_1}{dt} = -\lambda_1 N_1 \qquad \text{... (i)}$$

$$\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2 \qquad \text{... (ii)}$$

$$\frac{dN_3}{dt} = \lambda_2 N_2 \qquad \text{... (iii)}$$

Solving equation (i) : $(N_1)_t = N_{10} e^{-\lambda_1 t}$ ... (iv)

where N_{10} is the number of atoms of nuclide 1, at time $t = 0$

From equation (ii), using (iii), we obtain

$$\frac{dN_2}{dt} = \lambda_1 N_{10}\, e^{-\lambda_1 t} - \lambda_2 N_2$$

Or, $$\frac{dN_2}{dt} + \lambda_2 N_2 = \lambda_1 N_{10} e^{-\lambda_1 t} \quad \text{... (iv\_a)}$$

Multiplying both sides of equation (iv-a) by $e^{\lambda_2 t}$, the integrating factor,

$$e^{\lambda_2 t}\left(\frac{dN_2}{dt} + \lambda_2 N_2\right) = \lambda_1 N_{10}\, e^{(\lambda_2 - \lambda_1)t}$$

Or, $$\frac{d}{dt}\left(N_2 e^{\lambda_2 t}\right) = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1)t}$$

Integrating,

$$N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} e^{(\lambda_2 - \lambda_1)t} + A \quad \text{... (v)}$$

Where A is the constant of integration,

Now at $t = 0, (N_2)_t = N_{20} = a$ constant. So, from equation (v)

$$A = N_{20} - \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}$$

$$\therefore \quad (N_2)_t = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) + N_{20} e^{-\lambda_2 t} \quad \text{... (vi)}$$

From equation (iii), using (vi), we obtain on integration,

$$(N_3)_t = \left(\frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10} - N_{20}\right) e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} N_{10}\, e^{-\lambda_1 t} + B \quad \text{... (vii)}$$

At $t = 0, (N_3)_t = N_{30}$. So, from equation (vii), we get

$$B = N_{30} + N_{20} + N_{10}$$

$$\therefore \quad (N_3)_t = N_{30} + N_{20}\left(1 - e^{-\lambda_2 t}\right) + N_{10}\left(1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t}\right) \quad \text{... (viii)}$$

Equations (vi), (ii) and (viii) therefore, constitute the solution of the problem.

In most cases, only the first nuclide is present at $t = 0$. So, $N_{20} = N_{30} = 0$. We thus have from (vi) and (vii).

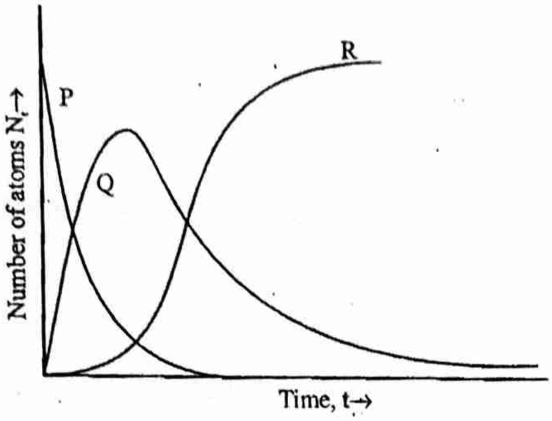
$$(N_2)_t = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_{10}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \quad \text{... (ix)}$$

$$(N_3)_t = N_{10}\left(1 + \frac{\lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t} - \frac{\lambda_2}{\lambda_2 - \lambda_1} e^{-\lambda_1 t}\right) \quad \text{... (x)}$$

The decay of the first nuclide P, the growth and decay of the second nuclide Q and the growth of the third nuclide R are shown in the figure below .

Time for Q(2) to attain a maximum:

Equation (ix) gives how N_2 varies with t. It shows that $N_2 = 0$ at $t = 0$. It increases with increasing t and attains a maximum at $t = t_m$, say.



Now, we have, $\frac{dN_2}{dt} = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}\left(-\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t}\right)$

$$\therefore \quad \frac{dN_2}{dt} = 0 \Rightarrow \lambda_2 e^{-\lambda_2 t_m} = \lambda_1 e^{-\lambda_1 t_m} \quad e^{(\lambda_1 - \lambda_2)t_m} = \frac{\lambda_1}{\lambda_2}$$

Or, $$(\lambda_1 - \lambda_2)t_m = \ell n \frac{\lambda_1}{\lambda_2}$$

$$\therefore \quad t_m = \frac{1}{\lambda_1 - \lambda_2} \ell n \frac{\lambda_1}{\lambda_2} \qquad \text{... (xi)}$$

That this t_m corresponds to maximum N_2 can be readily verified by obtaining the second time derivative of N_2 which becomes negative at $t = t_m$.

Branching of radioactivity :

When a radio-element shows that α and β activities, there are definite probabilities of α and β decay constants for the branches. It then be N atom, the total rate of disintegration will be

$$\frac{dN}{dt} = -\left(\lambda_\alpha + \lambda_\beta\right)N = \lambda N \qquad \text{where, } \lambda = \lambda_\alpha + \lambda_\beta \text{ the total decay constant.}$$

If $(T_{½})_\alpha$, $(T_{½})_\beta$ be the partial half lives of α and β decay.

$$\lambda_\alpha = \frac{0.693}{(T_{½})_\alpha}, \quad \lambda_\beta = \frac{0.693}{(T_{½})_\beta}$$

$$\therefore \quad \lambda = 0.693\left(\frac{1}{(T_{½})_\alpha} + \frac{1}{(T_{½})_\beta}\right)$$

But if $T_{½}$ be the mean half-life of the substance, then

$$\lambda = \frac{0.693}{T_{½}}$$

$$\therefore \quad \boxed{\frac{1}{T_{½}} = \frac{1}{(T_{½})_\alpha} + \frac{1}{(T_{½})_\beta}}$$

Determination of constants of radioactivity:

1. For simple decays : We have seen that the activity A of a sample exponentially with time according to the equation,

$$A_t = A_0\, e^{-\lambda t}$$

$\therefore$ $$\ell n\, A_t = \ell n A_0 - \lambda t$$

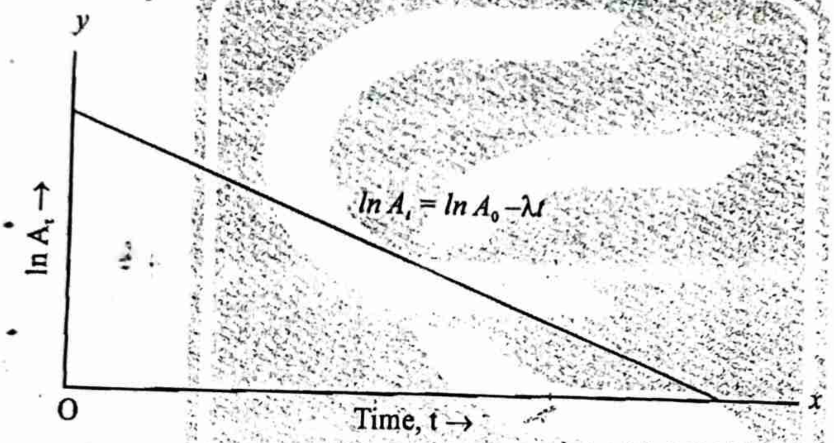
Or, $$y = -\lambda t + C \qquad \text{... (i)}$$

where, $y = \ell n\, A_t$ and $C = \ell n\, A_0$

Differentiating (i),

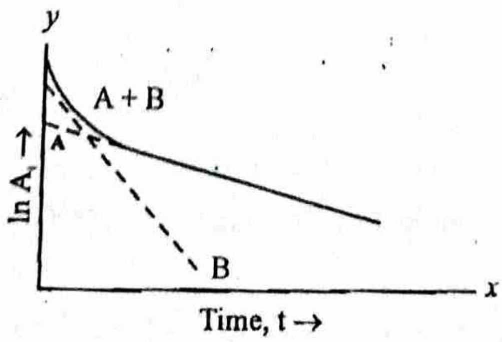
$$\frac{dy}{dt} = -\lambda$$

So, when the logarith of the activity, $\ln A_t$, is plotted against time t, it will give a straight line with a slope equal to $-\lambda$. This therefore would furnish us a method for evaluating λ, provided we can measure the activity of the sample.



2. Complex decays:

We conclude the section by indicating how one can determine half-lives for complex decays, e.g. two half-lives when a sample contains a mixture of say two activities A and B such that $T_{½}(A) >> T_{½}(B)$. A graph of the logarithm of the observed activity of the sample is first plotted against time, the observations however are extended over a time large enough compared to $T_{½}(B)$. The tail-end of the curve is due to the activity A, since the contribution from B in this region is negligible. So, if the straight part of the graph is extrapolated to $t = 0$, the contribution to the total observed activity of the sample from A at any instant t can be found. Subtracting this activity of A from the observed total activity at t, we get the activity of B at t. Once the contributions from the two activities are known, their half-lives can be evaluated by the usual procedure.



- **α-decay:**
  - **Velocity of α-particles:**

Let 'v' be the velocity of the $\alpha-$particles, M be its mass, q the charge, 'r' the radius of the track and B the magnetic field, then $v = Bqr/M$.

  - **Range of α-particles:**

The distance through which an $\alpha-$particle travels in a specified material before stopping to ionise it is called the range of α-particles in that material. The range is thus a sharply defined ionisation path-length.

The range depends on (i) The initial energy of the $\alpha-$particle; (ii) the ionisation potential of the gas and (iii) the chances of collision between the $\alpha-$particles and on the nature and the temperature and pressure of the gas. With increase of pressure, the range decreases; it increases if the temperature of the gas increased.

The range R in standard air is proportional to v^3, i.e. $R \propto v^3 \Rightarrow R = av^3$

The relation is known as the Geiger law, which is valid only in a limited velocity range.

Since $R \propto v^3$ and the energy $E = \frac{1}{2}mv^2$, the range-energy relationship is

$$R \propto E^{3/2} \Rightarrow R = bE^{3/2}$$

  - **Specific ionisation:**

The number of ion-pairs formed per unit path-length at any point in the path of the $\alpha-$particle is called specific ionisation and is symbolised by I.

Since, $$E \propto R^{2/3} \Rightarrow \frac{dE}{dR} \propto R^{-\frac{1}{3}} \propto \frac{1}{v}$$

  - **Geiger-Nuttal law:**

An important quantitative relation between the range R of the $\alpha-$particles and the decay constant λ of the emitting nuclei was experimentally discovered by Geiger and Nuttall(1911) and is called the Geiger-Nutall law. The relation runs as:

$$\ell n\,\lambda = A + B\,\ell n\,R$$

where A and B are constants having values different for different radioactive series.

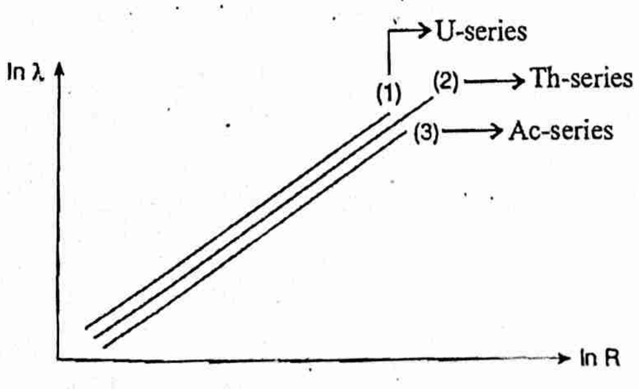


Figure: Variation of in λ with *l*nR: Geiger-Nuttall law

Since $R \propto E^{3/2} \Rightarrow \ell n\,\lambda = C + D\,\ell n\,E$

where C, D are two constants.

- α **-disintegration energy:**

$${}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

The Q-value of the decay process is known as the α – disintegration energy which is the total ener released in the disintegration process and is given by $Q_\alpha = (M_X - M_\alpha - M_Y)c^2$ where M's are the masses of the particles and 'c' the velocity of light in vacuum.

For heavy nuclei, Q_α is positive, so the decay can occur spontaneously as it does. According to la of conservation of momentum and energy.

$$0 = M_\alpha v_\alpha - M_Y v_Y$$

and $$Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y v_Y^2$$

$$\Rightarrow \quad Q_\alpha = \frac{1}{2}M_\alpha v_\alpha^2\left(1+\frac{M_\alpha}{M_Y}\right) = T_\alpha\left(1+\frac{M_\alpha}{M_Y}\right)$$

Kinetic energy of ejected α – particles, $T_\alpha = \dfrac{Q_\alpha}{1+M_\alpha/M_Y}$

$$\Rightarrow \quad Q_\alpha = T_\alpha \frac{[M_y + M_\alpha]}{M_y} = T_\alpha\left[\frac{A}{A-4}\right]$$

$$\Rightarrow \quad \boxed{T_\alpha = Q_\alpha\left[1-\frac{4}{A}\right]} \quad \& \quad T_d = Q_\alpha - T_\alpha = \frac{4Q}{A}$$

Where K.E. of α-particle = T_α

As $A \sim 200 \quad \Rightarrow \quad T_\alpha \sim Q_\alpha$

α- decay through tunnelling:

$${}_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + 2He^4$$

$$R = R_0 A^{1/3}$$

$$V(R) = \frac{2(Z-2)|e|^2}{4\pi \in_0 R}, \quad v = \sqrt{\frac{2E}{M}}$$

$$T = e^{-G}, G = A(Z-2)E^{-1/2} - B(Z-2)^{1/2} R^{1/2}$$

$$A = 1.587\times10^{-6}\ J^{1/2}$$

$$B = 94\times10^{6}\ m^{-1/2}$$

$$\nu = \frac{v}{2R};\ \lambda = \nu T;\ T_{1/2} = \frac{\ln 2}{\lambda}$$

Example: ${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$ and the energy of the emitted α particle is $E = 6.72\times10^{-13}$ *joule*. Determine the half-life time of ${}_{92}U^{238}$.

energy **Soln.** Nuclear radius of parent

$$R = R_0 A^{1/3} = (1.4\,fm)(238)^{1/3} = 8.7\,fm = 8.7\times10^{-15}\,m$$

Atomic number of parent Z = 92

to laws Tunnelling probability,

$$T = e^{-G} = e^{-\left(1.587\times10^{-6}\,J^{1/2}\right)(92-2)\left(6.72\times10^{-13}\,J\right)+\left(94\times10^{6}\,m^{-1/2}\right)(92-2)^{1/2}\left(8.7\times10^{-15}\,m\right)^{1/2}} = \frac{6}{10^{39}}$$

Out of 10^{39} strikes α emerges out six times only.

Velocity, $$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\left(6.72\times10^{-13}\,J\right)}{4\times1.67\times10^{-27}\,kg}} = 1.42\times10^{7}\,ms^{-1}$$

Collision frequency, $$\nu = \frac{v}{2R} = \frac{1.42\times10^{7}\,ms^{-1}}{2\left(8.7\times10^{-15}\,m\right)} = 8.2\times10^{20}\,s^{-1}$$

Decay constant, $$\lambda = \nu T = \left(8.2\times10^{20}\,s^{-1}\right)\left(6\times10^{-39}\right) = 4.92\times10^{-18}\,s^{-1}$$

Half life $$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{4.92\times10^{-18}\,s^{-1}} = 1.4\times10^{17}\,s$$

$$= \frac{1.4\times10^{17}}{365\times24\times60\times60}\,years = 4.4\times10^{9}\,years$$

The experimental value of half life is 4.5×10^{9} years. So, we have obtained a rough estimate of half life of ${}_{92}U^{238}$.

- **β – decay:**

Let 'v' be the velocity of a given β – particle, B the magnetic flux density, 'm' the relativistic mass of the β – particle and 'r' the radius of the circular track.

Then $$\frac{mv^2}{r} = Bev \quad \Rightarrow \quad r = \frac{mv}{Be} = \frac{m_0}{\sqrt{1-v^2/c^2}}\cdot\frac{v}{Be}$$

- β – decay, we write:

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e$$

The disintegration energy in β^- decay is:

$$Q_{\beta^-} = \left[M_n(A,Z) - M_n(A,Z+1) - m_e\right]c^2$$

$$= \left[M(A,Z) - Zm_e - M(A,Z+1) + (Z+1)m_e - m_e\right]c^2 \text{ (in terms of atomic mass)}$$

$$= \left[M(A,Z) - M(A,Z+1)\right]c^2$$

where M_n is the nuclear mass, M the atomic mass and m_e the mass of electron.

$$Q_{\beta^-} > 0, \text{ if } M(A, Z) > M(A, Z+1)$$

implying that β^- decay occurs only if the mass of the parent atom is greater than that of the daughter atom.

- **β^+ - decay:**

$$^A_ZX \rightarrow {}^A_{Z-1}Y + {}^0_{+1}e$$

$$\Rightarrow \quad Q_{\beta^+} = \left[M_n(A,Z) - M_n(A,Z-1) - m_e\right]c^2$$

$$= \left[M(A,Z) - Zm_e - M(A,Z-1) + (Z-1)m_e - m_e\right]c^2$$

$$= \left[M(A,Z) - M(A,Z-1) - 2m_e\right]c^2$$

For β^+ - decay to occur: $Q_{\beta^+} > 0, \text{ if } M(A,Z) > M(A,Z-1) + 2m_e$

i.e. the mass of the parent atom is greater than the daughter atom at least twice the electronic mass, i.e. 1.02 MeV.

- **Electron capture:**

$$^A_ZY + {}^0_{-1}e \rightarrow {}^A_{Z-1}Y$$

Therefore, disintegration energy, $Q_e = \left[M_n(A,Z) + m_e - M_n(A,Z-1)\right]c^2 - B_e$ where B_e is the binding energy of the electron to the orbit.

$$\Rightarrow \quad Q_e = \left[M(A,Z) - Zm_e + m_e - M(A,Z-1) + (Z-1)m_e\right]c^2 - B_e$$

$$= \left[M(A,Z) - M(A,Z-1)\right]c^2 - B_e$$

For electron capture to occur: $Q_e > 0, \text{ if } M(A,Z) > M(A,Z-1) + B_e$

i.e. the mass of the parent atom is greater than that of the daughter atom by at least the binding energy of the electron.

- **Selection rule for β^+ - decay:**

If ℓ_β is odd, initial and final nuclei must have opposite parities (parity changes in these transitions); for even ℓ_β values the initial and final nuclei must have same parity (no change in parity). Furthermore, as in allowed transitions, the emission of leptons (electron and neutrino) in the singlet state (Fermi-selection rule) requires $\Delta I \leq \ell_\beta$, whereas triplet-state (G-T selection rule) emission requires $\Delta I \leq \ell_\beta + 1$. Thus selection rules for forbidden transitions are:

First forbidden – For these transitions $\ell_\beta = 1$ and parity changes.

Fermi-selection rules: $\Delta I = \pm 1, 0 (\text{except } 0 \rightarrow 0)$

Gamow Teller rules: $\Delta I = \pm 2, \pm 1, 0 \left(\text{except } 0 \rightarrow 0, \frac{1}{2} \rightarrow \frac{1}{2}, 0 \leftrightarrow 1\right)$

The examples are:

${}^{39}_{18}Ar \to {}^{39}_{19}K + \beta^-$	$(7/2^- \to 3/2^+)$	*GT.*
${}^{14}_{6}C \to {}^{14}_{7}N + \beta^-$	$0 \to 1$	*F*
${}^{38}_{17}Cl \to {}^{38}_{18}Ar + \beta^-$	$(2^- \to 2^+)$	*F + GT mixed*
${}^{139}_{56}Ba \to {}^{139}_{57}La + \beta^-$	$(7/2^- \to 7/2^+)$	*F + GT mixed*
${}^{143}_{59}Pr \to {}^{143}_{60}Nd + \beta^-$	$(7/2^+ \to 7/2^-)$	*F + GT mixed*
${}^{147}_{61}Pm \to {}^{147}_{62}Sm + \beta^-$	$(7/2^+ \to 7/2^-)$	*F + GT mixed*
${}^{85}_{36}Kr \to {}^{85}_{37}Rb + \beta^-$	$(9/2 \to 5/2)$	*GT.*
${}^{89}_{38}Sr \to {}^{89}_{39}Y + \beta^-$	$(5/2^+ \to 1/2^-)$	*GT.*
${}^{137}_{55}Cs \to {}^{136}_{56}Ba^* + \beta^-$	$(7/2^+ \to 11/2^-)$	*GT.*

Second forbidden. For these transition $\ell_\beta = 2$ and no change in parity.

Fermi-selection rules: $\Delta I = \pm 2, \pm 1$ (except $0 \leftrightarrow 0$)

Gamow Teller rules: $\Delta I = \pm 3, \pm 2$ (except $0 \leftrightarrow 2$)

The examples are:

${}^{137}_{55}Cs \to {}^{137}_{56}Ba + \beta^-$	$(7/2^+ \to 3/2^+)$	*F + GT mixed*
${}^{10}_{4}Be \to {}^{10}_{5}B + \beta^-$	$(0^+ \to 3^+)$	*GT.*
${}^{22}_{11}Na \to {}^{22}_{10}Ne + \beta^+$	$(3^+ \to 0^+)$	*GT.*

- **γ-decay:**
  In it no change in Z and A. γ-ray photons are emitted when the nucleus jumps from an excited state to the lower state (after 10^{-14} sec). It is always not possible that when nucleus jumps from higher energy state to lower γ-photon will be emitted.

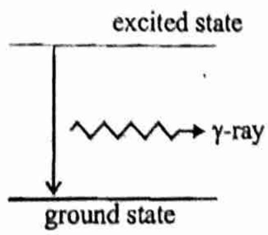
  Sometimes the nucleus in higher energy state directly give energy to the atomic e^- and hence an e^- may come out rather than a γ-ray photon. This process is known as internal conversion and the e^- so emitted is called as conversion e^-.

Energy of emitted γ-ray in α-decay:

Let I have a parent nuclei ${}^{A}X_{Z}$, when it emit a particle it goes to the excited state and it comedown to its ground state by emitted γ-ray.

According energy conservation rule we can write,

$$E_\gamma = Q_\alpha - (T_\alpha + T_y) = Q_\alpha - T_\alpha\left(1 + \frac{M_\alpha}{M_y}\right)$$



$$\therefore \quad \boxed{E_\gamma = Q_\alpha - T_\alpha\left(1 + \frac{M_\alpha}{M_y}\right)}$$

Type	Symbol	Change in Angular momentum, L	Parity change
Electric dipole	E_1	1	Yes
Magnetic dipole	M_1	1	No
Electric quadrupole	E_2	2	No
Magnetic quadrupole	M_2	2	Yes
Electric octupole	E_3	3	Yes
Magnetic octupole	M_3	3	No
Electric 2^L – pole	E_L	L	[No for L even. Yes for odd]
Magnetic 2^L – pole	M_L	L	[Yes for L even. No for L odd]

γ-ray interaction with matter :

There are three fundamental procedure in which γ-ray interact with matter. These three processes

(i) Compton scattering

(ii) Photoelectric effect

(iii) Pair production

The three factors should therefore be combined into one single (total) absorption coefficient, μ in the absorption law. The intensity of the transmitted γ-ray is given by

$$\boxed{I = I_0 e^{-\mu x}}$$ where, x is the thickness of medium.

SOLVED PROBLEMS

1. A nucleus having mass number 240 decays by a emission to the grond state of its daughter nucleus. The Q value of the process is 5.26 MeV. The energy (in MeV) of the a particle is: **[GATE 2005]**

(a) 5.26 (b) 5.17 (c) 5.13 (d) 5.09

Soln. $M_x = 240 \quad Q = 5.26\ MeV$

We know,

$$T_\alpha = Q\left[1-\frac{4}{M_x}\right] = Q\left[1-\frac{4}{240}\right] = 5.26\left[1-\frac{1}{60}\right] = 5.26\times\left(\frac{59}{60}\right) = 5.1723$$

Correct option is (b)

2. Which one of the following disintegration series of the heavy elements will give ^{209}Bi as a stable nucleus? **[GATE 2006]**

(a) Thorium series (b) Neptunium series

(c) Uranium series (d) Actinium series

Soln. We know that,

(i) Thorium series is 4n series (ii) Uranium series is 4n + 2 series.

(iii) Actinium series is 4n + 3 series. (iv) Nepturium series is 4n + 1 series.

Therefore, ^{209}Bi is the stable element of Nepturium series.

Correct option is (b)

3. Fission fragments are generally radioactive as: **[GATE 2007]**

(a) they have excess in neutrons

(b) they have excess of protons

(c) they are products of radioactive nuclides

(d) their total kinetic energy is of the order of 200 MeV

Soln. The condition for radioactivity $\frac{n}{z} \geq 1.5$

Therefore, the fission fragments are generally radioactive because they have excess in neutrons.

Correct option is (a)

4. Half life of a radio-isotope is 4×10^8 years. If there are 10^3 radioactive nuclei in a sample today, the number of such nuclei in the sample 4×10^9 year ago were: **[GATE 2007]**

(a) 128×10^3 (b) 256×10^3 (c) 512×10^3 (d) 1204×10^3

Soln. $T_{½} = 4\times10^8$ *years* Therefore, number of half life time $\frac{4\times10^9}{4\times10^8} = 10$

Therefore, remaining number of particle after 10 half life time $N = \frac{N_0}{2^{10}}$

$$N_0 = N\times2^{10} = 1024\times10^3$$

Correct option is (d)

5. The disintegration energy is defined to be the difference in the rest energy between the initial and final states. Consider the following process: **[GATE 2009]**

$$^{240}_{94}Pu \rightarrow ^{236}_{92}U + ^{4}_{2}He$$

The emitted α particle has a kinetic energy 5.17 MeV. The value of the disintegration energy is

(a) 5.26 MeV (b) 5.17 MeV (c) 5.08 MeV (d) 2.59 MeV

Soln. We know that, $T_a = Q\left(1-\frac{A}{M_x}\right)$

$$\Rightarrow \quad Q = \frac{T_a}{1-\frac{A}{M_x}} = \left(\frac{5.17}{1-\frac{4}{240}}\right) = 5.17\left(1+\frac{4}{240}\right) = 5.17\left(\frac{244}{240}\right) = 5.26\ MeV$$

Correct option is (a)

6. ^{238}U decays with a half life of 4.51×10^9 years, the decay series eventually ending at ^{206}Pb, which is stable. A rock sample analysis shows that the ratio of the numbers of atoms of ^{206}Pb to ^{238}U is 0.0058. Assuming that all the ^{206}Pb has been produced by the decay of ^{238}U and that all other half-lives in the chain are negligible, the age of the rock sample is **[JEST 2013]**

(a) 38×10^6 years (b) 48×10^6 years (c) 38×10^7 years (d) 48×10^7 years

Soln. $T_{½} = 5.51\times10^9$, $\quad \lambda = \frac{0.693}{4.51\times10^9}\ years^{-1}$

$\frac{Pb}{U_{re}} = 0.0058 \quad Pb = (U_{re}\times0.0058)$

Therefore, there was total number of uranium atom = Pb + U_{re} = 1.0058 U_{re}.

$$\therefore \quad \frac{U_{re}}{U} = e^{-\lambda t} \quad \Rightarrow \frac{1}{1.0058} = e^{-\lambda t}$$

$$\Rightarrow \quad t = -\frac{1}{\lambda}\ell n\left(\frac{1}{1.0058}\right) = -\frac{4.51\times10^9}{0.693}\times\ell n\left(\frac{1}{1.0058}\right)$$

$$= \frac{4.51\times0.005783\times10^9}{0.693} \simeq 38\times10^6\ years$$

Correct option is (a)

7. In the β decay process, the transition $2^+ \to 3^+$, is **[GATE 2013]**
   (a) allowed both by Fermi and Gamow-Teller selection rule
   (b) allowed by Fermi and but not by Gamow-Teller selection rule
   (c) not allowed by Fermi but allowed by Gamow-Teller selection rule
   (d) not allowed both by Fermi and Gamow-Teller selection rule

Soln. According to Fermi-Selection rules $\Delta I = \pm 1$ $\quad (\text{except } 0 \to 0)$

Gamow Teller rules $\Delta I = \pm 2, \pm 1, 0 \left(\text{except } 0 \to 0, \frac{1}{2} \to \frac{1}{2}, 0 \leftrightarrow 1\right)$

Therefore, the transition $2^+ \to 3^+ \quad \Delta I = +I$

Therefore, allowed both by fermi and Gamow-Teller selection rule.

Correct option is (a)

8. Which of the following radioactive decay chains is possible to observe ? **[TIFR 2015]**
   (a) ${}^{206}_{82}\text{Pb} \to {}^{202}_{80}\text{Hg} \to {}^{202}_{79}\text{Au}$ (b) ${}^{210}_{83}\text{Bi} \to {}^{210}_{84}\text{Po} \to {}^{206}_{82}\text{Pb}$
   (c) ${}^{214}_{88}\text{Ra} \to {}^{210}_{86}\text{Rn} \to {}^{207}_{82}\text{Pb}$ (d) ${}^{206}_{82}\text{Pb} \to {}^{202}_{80}\text{Hg} \to {}^{202}_{79}\text{Au}$

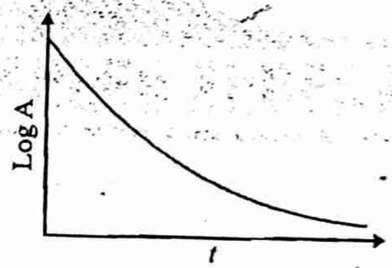
Soln. We know when α - decay take place mass number and atomic number decrease by 4 and 2 but for β^- decay atomic number increase by 1.

Only the option to (b) satisfy the condition

$${}^{210}_{83}Bi \xrightarrow{\beta} {}^{210}_{84}P_0 \xrightarrow{\alpha} {}^{206}_{82}Pb$$

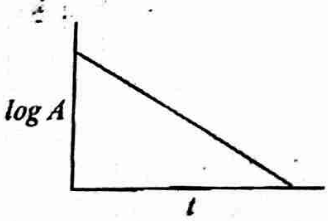
Correct option is (b)

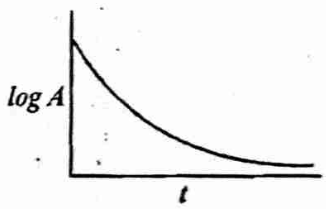
9. The plot of $\log A$ vs. time t, where A is activity, as shown in the figure, corresponds to decay **[GATE 2005]**



(a) from only one kind of radioactive nuclei having same half life
(b) from only neutron activated nuclei
(c) from a mixture of radioactive nuclei having different half lives
(d) which is unphysical.

Soln. For one kind of radioactive nuclei the graph between $\log A$ and t become straight line.
But for mixture of radioactive nuclei it become curve





Correct option is (c)

10. Radioactive sample has its half-life equal to 60 days. Calculate its (i) disintegration constant, (ii) Its average life, (iii) the time required for 2/3 of the original number of atoms to disintegrate and (iv) the time taken for 1/4 of the original number of atoms to remain unchanged.

Soln. (i) Since $t_{1/2} = 60$ days, $\lambda = 0.693/t_{1/2} = 0.693/60 = 0.01155\ \text{day}^{-1}$

(ii) Since $\lambda = 0.01155\ \text{day}^{-1}$, $\overline{T} = 1/\lambda = 1/0.01155 = 86.58$ days

(iii) Number to be disintegrated $= \frac{2}{3}N_0$. So, the number to remain unchanged $= \frac{1}{3}N_0$

Thus, $N/N_0 = \frac{1}{3}$

Therefore, From the relation $N = N_0 e^{-\lambda t}$, we obtain $\frac{1}{3} = e^{-\lambda t} \Rightarrow \lambda t = \ell n\, 3$

$$\Rightarrow \quad t = \frac{\ell n\, 3}{\lambda} = \frac{2.3026 \times 0.4771}{0.01155} = 95.1 \text{ days}$$

(iv) Here $N/N_0 = \frac{1}{4}$. So, from the decay law, $\frac{1}{4} = e^{-\lambda t} \Rightarrow \lambda t = \ell n\, 4$

$$\Rightarrow \quad t = \frac{\ell n\, 4}{\lambda} = \frac{2.3026 \times 0.6021}{0.01155} = 120 \text{ days}$$

11. (a) A radioactive substance disintegrates for a time equal to its average life. Calculate the fraction of the original substance disintegrated.
(b) The half-life of a radon is 3.82 days. What fraction of freshly prepared sample of radon will disintegrate in 10 days?

Soln. (a) Here $\overline{T} = t = \frac{1}{\lambda}$. Since, $\frac{N}{N_0} = e^{-\lambda t} = e^{-\lambda \overline{T}} = e^{-1} = 0.368$

Therefore, Fraction disintegrated $= 1 - 0.368 = 0.632$

(b) Here $T_{1/2} = 3.82$ days and $t = 10$ days. Therefore, we have $\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{3.82} = 0.181\ \text{day}^{-1}$

Now, $\frac{N}{N_0} = e^{-\lambda t} = e^{-0.181 \times 10} = e^{-1.81} = \frac{3}{20}$

$$\Rightarrow \quad \text{Fraction disintegrated} = 1 - \frac{3}{20} = \frac{17}{20}$$

12. The half-life of UX_1 is 24.1 days. How many days, after UX_1 has been isolated, will it take for 90% of it to change to UX_2?

Soln. Here $T_{1/2} = 24.1$ days $\Rightarrow \lambda = 0.693/24.1 = 0.0287\ \text{day}^{-1}$

Amount disintegrated $= 90/100 \Rightarrow N/N_0 = 10/100 = 1/10$

Since, $\frac{N}{N_0} = e^{-\lambda t}$, $\frac{1}{10} = e^{-0.0287 t}$

$$\Rightarrow \quad t = \frac{\ell n\, 10}{0.0287} = \frac{2.3026 \times 1}{0.0287} = 80 \text{ days}$$

13. The half-life of radioactive K-40 is 1.83×10^{8} years. Find the number of $\beta-$particles emitted per sec per g. of K-40, assuming $\lambda=1.2\times10^{-17}\ s^{-1}$, Avogrado number = 6.02×10^{23}.

Soln. Let N_t = Number of atoms of K-40 in 1g at t = $6.02\times100^{23}/40$

$$\Rightarrow \quad \left|\frac{dN_t}{dt}\right|=\lambda N_t=\frac{1.2\times10^{-17}\times6.02\times10^{23}}{40}=1.8\times10^{5}$$

So, the number of particles emitted per g of K-40 is 1.8×10^{5}

14. It is observed that 3.67×10^{10} $\alpha-$particles are emitted per g of Ra- 226. Calculate the half-life of Ra-226. Avogadro number = 6.023×10^{23}.

Soln. 1g of Ra-226 = $6.023\times10^{23}/226$ atoms of Ra-226. Of these 3.67×10^{10} disintegrate per sec. So, the decay constant.

$$\lambda=\frac{3.67\times10^{10}}{6.023\times10^{23}/226}\ s^{-1} \Rightarrow T_{1/2}=\frac{0.693}{\lambda}=\frac{0.693\times6.023\times10^{23}}{3.67\times10^{10}\times226}\ s=1595 \text{ years}$$

15. A large amount of radioactive material of half-life 20 days got spread in a room making the level of radiation 40 times the permissible level of normal occupancy. After how many days would the room be safe for occupation.

Soln. Let after 't' days the room would be safe for occupation. So, in 't' days, the activity would drop down to 1/40 of its initial value.

$$\Rightarrow \quad \frac{N}{N_0}=\frac{1}{40}. \text{ From the relation } \frac{N}{N_0}=e^{-\lambda t},\ \frac{1}{40}=e^{-(0.693/20)t}$$

$$\left(\because\ T_{1/2}=40 \text{ days},\ \lambda=(0.693/20)\,\text{day}^{-1}\right) \Rightarrow \ell n\, 40=\frac{0.693}{20}t$$

$$\Rightarrow \quad t=\frac{2.303\times20}{0.693}\log_{10}40=\frac{2.303\times20\times1.6021}{0.693}=106.4$$

Therefore, room would be safe for occupation after 107 days.

16. Calculate the amount of Ra-226 in secular equilibrium with 1 kg of pure U-238, given the half-lives of Ra-226 and U-238 as 1620 years and 4.5×10^{9} years respectively.

Soln. Let x in g be the required amount of Ra-226 in secular equilibrium with 1 kg of pure U-238. Now:

$$1 \text{ kg of pure U-238} = \frac{6.02\times10^{23}}{238}\times10^{3} \text{ atoms of U}$$

$$x \text{ g of Ra-226} = \frac{6.02\times10^{23}}{226}\times x \text{ atoms of Ra}$$

The condition of secular equilibrium gives, $\dfrac{N_U}{(T_{1/2})_U}=\dfrac{N_{Ra}}{(T_{1/2})_{Ra}}$

$$\Rightarrow \quad \frac{6.02\times10^{23}\times10^{3}}{238\times4.5\times10^{9}}=\frac{6.02\times10^{23}\times x}{226\times1620} \quad \Rightarrow \quad x=\frac{226\times1620\times10^{3}}{238\times4.5\times10^{9}}=34.18\times10^{-5}\ g=0.34\ mg$$

sec per 17. The Half life of $_{92}U^{238}$ is 4.51×10^9 yrs. What percentage age of $_{92}U^{238}$ that existed 10^{10} years ago still survives.

Soln. $\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{4.5 \times 10^9}$

If N_0 = number of atom of $_{92}U^{238}$ excited 10^{10} year ago $N = N_0$.

Now present, $N = N_0 e^{-\lambda t}$ where $t = 10^{10}$ yrs $\Rightarrow \frac{N}{N_0} = e^{\lambda t}$ or $\log_e \frac{N_0}{N} = \lambda t$

e of Ra- $\Rightarrow 2.3026 \log_{10} \frac{N_0}{N} = \lambda t = \frac{0.693 \times 10^{10}}{4.51 \times 10^9} \Rightarrow \log_{10} \frac{N_0}{N} = \frac{0.693 \times 10}{2.3026 \times 4.51} = 0.6673$

ne decay $\Rightarrow \frac{N_0}{N} = \text{anti}\log 0.6673 = 4.648 \Rightarrow \frac{N}{N_0} = 0.215$

s % of $_{92}U^{235}$ now present $= 0.215 \times 100 = 21.5\%$

18. The half life of a radioactive substance is 5 hr. What will be its one third life time?

radiation **Soln.** $T_{1/2} = 5$ hrs. $T_{1/2} = \frac{0.693}{\lambda} \Rightarrow \lambda = \frac{0.693}{5} = 0.1386$ per hour also $\frac{N}{N_0} = e^{-\lambda t}$.

e safe fo

op down In this case $\frac{N}{N_0} = \frac{1}{3} \Rightarrow \frac{1}{3} = e^{-\lambda t} \Rightarrow 3 = e^{\lambda t}$

Hence, $\log_e 3 = \lambda t$ or $t = \frac{2.3026 \log_{10} 3}{0.1386} = 7.93$ hrs

19. The activity of certain radio nuclide decreases to 15% of its original value in 10 days. Find its half life.

Soln. Let N_0 be the original no. of nuclei and N left behind after 10 days. If λ is the radioactive constant, then

$\Rightarrow \frac{N}{N_0} = e^{-\lambda t} \Rightarrow \frac{15}{100} = e^{-\lambda 10}$

$\Rightarrow \log_e \frac{100}{15} = 10\lambda \Rightarrow \lambda = \frac{1}{10} \log_e \frac{100}{15} = \frac{1}{10} \times 2.3026 \log_{10} \frac{100}{15} = 0.1897$

$\Rightarrow T_{1/2} = \frac{0.6931}{\lambda} = \frac{0.6931}{0.1897} = 3.65$ day

of Ra

Now:

.34 mg

PRACTICE SET

1. Which of the following phenomena can be explained by quantum mechanical tunneling through the barrier?

(i) α – Decay (ii) Field emission of e^{-s} from a metal surface. (iii) β – Decay

Select the correct answer using the code given below:

(a) i, ii, iii (b) i only (c) i & ii only (d) ii & iii only.

2. ^{10}Be in its first excited state has spin-parity 2^+. It gets de-excited to the ground state, which has spin parity 0, by γ-emission. The multipoles carrried by γ are

(a) E2 (b) M2 (c) E2, M2 (d) E4.

3. Which of the following is true for β-decay of the neutron? The process

(a) Violates both parity and charge conjugation symmetry.
(b) Violates parity but conserves charge conjugation symmetry.
(c) Conserves parity but violates charge conjugation symmetry.
(d) Conserves both parity and charge conjugation symmetry.

4. Consider the alpha-decay reaction, $Po_{84}^{210} \rightarrow Pb_{82}^{206} + He_2^4$ where atomic masses are

$m(P_0) = 210.0483$ amu; $m(Pb) = 206.0386$ amu

$m(He) = 4.0039$ amu; 1 amu = 931.141 MeV

The kinetic energy of the alpha particle will be

(a) 5.4 keV (b) 2.7 keV (c) 5.4 MeV (d) 10.8 MeV

5. The energy spectrum of beta-particles is continuous because

(a) parity is violated in beta-decay
(b) beta-particles are emitted in the continuum region
(c) beta- decay leads to a three body state
(d) beta-decay is a weak interaction process

6. The $0 \rightarrow 0$ gamma transition is forbidden because

(a) photon has integer spin
(b) photon is a massless particle
(c) photon has a definite polarization
(d) parity will not be conserved

7. If decay constant of a radioactive sample is λ per minute, then the fraction that decays in the fifth minute is

(a) $e^{-4\lambda}$ (b) $e^{-5\lambda}$ (c) $e^{-4\lambda} - e^{-5\lambda}$ (d) $e^{-4\lambda} + e^{-5\lambda}$

8. In the β decay process, the transition $2^+ \rightarrow 3^+$, is

(a) allowed both by Fermi and Gamow-Teller selection rule
(b) allowed by Fermi and but not by Gamow-Teller selection rule
(c) not allowed by Fermi but allowed by Gamow-Teller selection rule
(d) not allowed both by Fermi and Gamow-Teller selection rule

9. The radioactive decay of a certain material satisfies Poisson statlstics with a mean rate of λ per second what should be the minimum duration of counting (in seconds) so that the relative error is less than 1%

(a) $100/\lambda$ (b) $10^4/\lambda^2$ (c) $10^4/\lambda$ (d) $1/\lambda$

10. Which of the following radioactive decay chains is it possible to observe?

(a) $^{206}_{82}Pb \rightarrow ^{202}_{80}Hg \rightarrow ^{202}_{79}Au$
(b) $^{210}_{83}Bi \rightarrow ^{210}_{84}Po \rightarrow ^{206}_{82}Pb$
(c) $^{214}_{88}Ra \rightarrow ^{210}_{86}Rn \rightarrow ^{207}_{82}Pb$
(d) $^{206}_{82}Pb \rightarrow ^{202}_{80}Hg \rightarrow ^{202}_{79}Au$

ANSWER KEY					
Questions	1	2	3	4	5
Option	(c)	(a)	(b)	(c)	(c)
Questions	6	7	8	9	10
Option	(d)	(c)	(a)	(c)	(b)

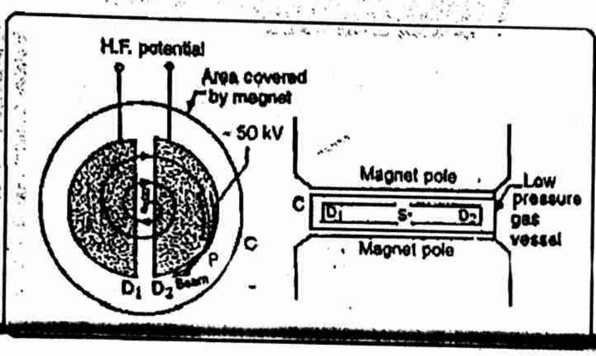
Particle Acceleration

Introduction :

The purpose of an accelerator of chraged particles is to direct against a target a beam of a specific kind of particles of chosen energy. There ae many varities of methods for accomplishing this task. All using various arrangements of electronic and magnetic fields, and this chapter we will read about some types of accelerators.

Cyclotron :

Construction: It consists essentially of two flat semi-circular hollow metal boxes D_1, D_2 which are called the dees (D) because of their shape (figure below). These hollow chambers have their diametric edges parallel and slightly separated from each other so as to produce a narrow gap between them. The dees are placed in an evacuated chamber C and are connected



to an rf-oscillator so that a high frequency (10 MHz) alternating potential is applied between the dees acting as electrodes. The potential between the dees thus alternates rapidly and the electric field in the gap is first directed to one dee and then to the other. The space within each dee is however field-free (zero left). The chamber C is mounted horizontally between the pole pieces of a huge electromagnet that provides an intense and uniform vertical magnetic field of several tesla. Since the field must be uniform over the whole of the dees, the diameter of the poels should be slightly greater than that of the dees, the diameter of the poles should be slightly greater than that of the dees.

The ion-source S is located near the mid-point of the gap between the gap between the dees. The ions, after being accelerated, are brought out of the chamber throught a window P by a charged deflecting plate to bombard the target.

Principles of action: Let a positive ion of charge q leave the ion-source between the dees and enter the negative dee at the instant with velocity $\vec{v}_0$, normal to the magnetic lines of force. Due to the magnetic field $\vec{B}$, the force acting on the particle is

$$\vec{F} = q\vec{v}_0 \times \vec{B} \qquad \text{... (1)}$$

But $\vec{B}$ is perpendicular to the force so that we obtain

$$\vec{F} = qv_0B \qquad \text{... (2)}$$

Under the action of the field, the path of the particle is bent into a circle of radius r_0, given by the relation,

$$qv_0B = \frac{mv_0^2}{r_0}$$

$$\therefore \qquad \boxed{r_0 = \frac{mv_0}{qB}} \qquad \text{... (3)}$$

where m is the mass of the ion (particle).

[redacted] inside the dee, as it leaves it to reach the gap, the electric field of the rf-source changes direction synchronously. The particle thus gets a 'kick' and is accelerated into the other dee where, because of its increased speed v, it moves in a semi-circular path of comparatively larger radius r, given by

$$\boxed{r = \frac{mv}{qB}} \qquad \text{... (4)}$$

to emerge again into the gap between the dees.

The frequency of revolution of the particle in the circular orbit is given by

$$f = \frac{v}{2\pi r} = \frac{vqB}{2\pi mv} = \left(\frac{q}{m}\right)\frac{B}{2\pi} \qquad \text{... (5)}$$

Using equation (4).

The important fact that emerges from (5) is that the frequency of revolution is independent of the speed of the ion and the radius of path in the dees. So, if the electric field reverses regularly at a frequency exactly [redacted] each time the gap is crossed.

$$\therefore \qquad \boxed{\left(\frac{q}{m}\right)\frac{B}{2\pi} = f = f'}$$

This is called the basic resonance equation for a fixed frequency cyclotron, f' being the frequency of the oscillator output.

Thus the revolving charged particle is steadily speeded up describing a flat spiral of increasing radius. Finally, it reaches the periphery of the dees and is brought out of the chamber through window by a negatively charged deflector to impinge on the properly mounted target. We shall now calculate the energy of the particle

Energy of the particle : The kinetic energy W_k of the charged particle is given by

$$W_k = \frac{1}{2}mv^2$$

[redacted]

$$W_m = \frac{1}{2}mv_m^2 = \frac{1}{2}m\left(\frac{r_mqB}{m}\right)^2, \text{ using (4)}$$

$$= \frac{q^2B^2}{2m}r_m^2 \qquad \text{... (7)}$$

Thus, the maximum energy that can be imparted to a particle is directly proportional to the square of (1) the radius of the dees and (2) the magnetic field of the magnet.

The energy of the particles expressed in equivalent voltage V is given by

$$\boxed{V = qB^2 r_m^2 / 2m} \quad \text{... (8)}$$

Betatron :

As already indicated, the maximum energy that could be imparted to a particle in a cyclotron is limited by [redacted] to accelerate electrons as their relativistic mass increase even at low energies is quite high. The above difficulty is overcome in a betatron designed and developed by D.W. Kerst in 1941 at the University of Illinois. In fact, the betatron was specifically meant for acceleration the electrons.

Basic principle : The basic principle of betatron is to accelerate electrons in a stable orbit of essentially constant radius by the application of an alternating magnetic field, called the induction field. The induction field mainly produces two effects.

(1) the increasing magnetic flux produces an electromagnetic force that accelerates the electrons along their orbits and thus increases their energy in each successive orbital revolution

(2) the varying magnetic field, acting perpendicular to the electron-orbit, simultaneously constrains the electrons move in a circualr trajectory of constant radius.

Betatron condition: It is possible to derive a condition for a stable orbit of constant radius in a betatron as follows.

[redacted]

$$\in = \frac{d\phi}{dt} \text{(save the sign)} \quad \text{... (1)}$$

where ϕ is the magnetic flux enclosed by the orbiting electrons.

If r be the radius of the electron orbit, the induced emf is also given by

$$\in = 2\pi r X \quad \text{... (2)}$$

where X is the induced electric field.

Therefore, the force acting on the electron of charge $\in$ is

$$F = eX = e\frac{\in}{2\pi r} = \frac{\in}{2\pi r} = \frac{e}{2\pi r}\frac{d\phi}{dt} \quad \text{using (1) and (2)} \quad \text{... (3)}$$

As the electron of mass m and velocity v is deflected by magnetic induction B along an orbit of radius r,

$$\frac{mv^2}{r} = Bev$$

Therefore, momentum, $p = mv = Ber$ ... (4)

$$\therefore \quad F = \frac{dp}{dt} = \frac{d}{dt}(Ber) = er\frac{dB}{dt}, \text{ using (6)} \quad \text{... (5)}$$

Comparing (2) and (5), $\boxed{\frac{d\phi}{dt} = 2\pi r^2 \frac{dB}{dt}}$ ... (6)

This equation is known as the betatron condition,

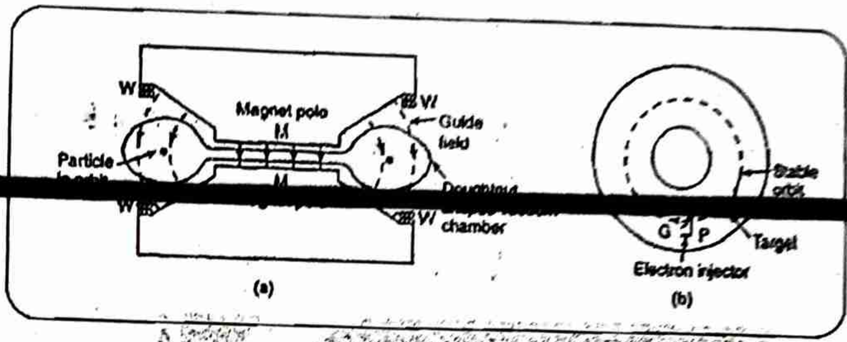
Integrating equation (6), $\phi = 2\pi r^2 B$ ... (7)

For a uniform magnetic induction, however, the flux would be $\pi r^2 B$

Thus for a stable orbit of constant radius, the pole pieces of the magnet are to be so designed that the flux must change at twice the rate at which it would change if the magnetic induction were uniform throughout the area enclosed by the orbit.

The above condition is achieved by designing pole pieces from laminated iron core in which the varying [redacted] magnetic field is produced by an a.c. supply of about 60 Hz. Then the flux density at the centre of the orbit is larger than that at the edges. This condition is valid for both relativistic and non-relativistic energies of the electron.

Construction: The betatron (figure below) consists essentially of
(i) a highly evacuated doughnut-shaped glass chamber containing the source of electrons;
(ii) a specially shaped powerful electromagnet MM between the pole piece of which the doughnut is mounted-the pole pieces being so designed that they have a strong field at the centre with gradient towards the edges;
(iii) the windings WW through which passes an alternating current to energies the



(a)

(b)

electromagnet,
(iv) the filament F which is the source of electrons produced thermoinically. The thermoelectrons are given an initial energy of 50 kV by applying a high potential between F and the plate P, G being the grid to focus the electrons. The electrons are injected into the doughnut at a time when the magnetic flux produced by the electromagnetic rises from zero in the first quarter cycle, T is the target which is impinged by the stream of accelerated electrons.

When the magnetic field increases from zero to the final positive maximum, the electrons complete enough number of revolutions (thousands) in the fixed stable orbit gaining higher and higher energies. When the desired energy is achieved, the radius of the orbit is suddenly increased by increasing the flux ϕ, without changing the field as given by the relation $r = (\phi / 2\pi B)^{1/2}$, and the electrons are deflected onto the target T. The above is ensured by the discharge of a condenser through the primary of the electromagnet that increases the flux without chaning the flux density

Energy computation: The momentum p imparted to an electron of charge e by a field B is given by

$$\frac{mv^2}{r} = Bev$$

Or, $$p = mv = Ber$$

Therefore, the energy (relativistically) is : $E = pc = Berc$
where B is the magnetic field at the end of the acceleration time.

In a betatron, the acceleration time is $\frac{1}{4}T$ where T is the period $\left(= \frac{2\pi}{\omega}\right)$, ω being the angular frequency.

$$\frac{T}{4} = \frac{1}{4} \cdot \frac{2\pi}{\omega} = \frac{\pi}{2\omega}$$

Therefore, distance travelled by the electron in time $\frac{T}{4}$ is $d = \frac{\pi c}{2\omega}$, since the velocity of electron approaches the velocity of light c after being accelerated.

Therefore, Number of revolutions executed per sec by the electron is

$$n = \frac{d}{2\pi r} = \frac{\pi c}{2\omega} \times \frac{1}{2\pi r} = \frac{c}{4\omega r} \qquad \ldots (9)$$

[redacted]

When the flux changes as a function of time, the energy gained per sec by the electron is

$$e\frac{d\phi}{dt} = e\frac{d}{dt}(\phi_0 \sin \omega t) \qquad [\because \phi = \phi_0 \sin \omega t]$$

$$= e\omega\phi_0 \cos \omega t$$

Since the acceleration occurs in time T/4, the average energy gained in each revolution is

$$\int_0^{T/4} e\omega\phi_0 \cos \omega t dt \Big/ \int_0^{T/4} dt = e\omega\phi_0 \int_0^{T/4} \cos \omega t dt \Big/ \int_0^{T/4} dt = \frac{2e\omega\phi_0}{\pi}$$

[since, $T/4 = \pi/2\omega$ by equation (8)]

Therefore, the total energy imparted to the electron in n revolution is

E_T = mean energy per rev. × number of revolutions

$= \frac{2e\omega\phi_0}{}$ × $\frac{c}{}$ = $\frac{ec\phi_0}{}$, using (9) [redacted]

Difference between Cyclotron and Betatron :

	CYCLOTRON		BETATRON
1.	Fixed-frequency cyclotron employs no alternating magnetic field; the magnetic field is constant.	1.	Betatron, however, employs an oscillating field which is called the induction field.
2.	Particles are accelerated in helical orbits and so orbital radius goes on increasing and equals the radius of the dees in the limit	2.	The acceleration of the particles in a betatron takes place in a circular orbit which is essentially of constant radius.
3.	A fixed-frequency cyclotron is unsuitable for accelerating electrons	3.	A betatron is specifically meant for accelerating electrons
[redacted]	[redacted] orbital stability.	[redacted]	[redacted] orbital stability
5.	This cannot be used as an X-ray generator	5.	This is a very highly efficient X-ray generator.

SOLVED PROBLEMS

[redacted]
of radius 100 mm is ________ Tesla. **[GATE 2014]**

(Given: $m_p = 1.67\times10^{-27}\,kg, e = 1.6\times10^{-19}\,C$

Soln. $\frac{1}{2}mv^2 = 1\times10^6\times1.6\times10^{-19}\,J$

$\Rightarrow \quad v = \sqrt{\frac{3.2\times10^{-13}}{m}} = \sqrt{\frac{3.2\times10^{-13}}{1.67\times10^{-27}}} = \sqrt{1.91677\times10^{14}} = 1.384474\times10^7\ m/\sec$

Now, for circular orbit

$$q\,vB = \frac{mv^2}{r}$$

[redacted]

$$qr \qquad 1.6\times10^{-19}\times10^{-1}$$

$$= \frac{1.67\times1.384474}{1.6} = 1.4450\ \text{Tesla}$$

2. Four partices-electron, proton, He^+ and Li^+ are projected in a circular orbit of same radius perpendicular to a given magnetic field. Then the velocity will be smallest for
(a) electron (b) He^+ (c) proton (d) Li^+

Soln. We know that radius of the circle

$$r_L = \frac{mv}{qB}$$

Since, q, B and r are fixed

$$\therefore \quad v \propto \frac{1}{m}$$

[redacted]

Correct option is (c)

3. Deuterons in a cycltron describe a circle of radius 0.32 m just before emerging from the dees. The ac voltage applied to the dees is 2×10^4 volt at 10 MHz. Find (i) the velocity of deutrons, (ii) the magnetic field and (iii) the energy of deutrons in MeV:

Soln. (i) From the relation $v = 2\pi rn$, the velocity is given by

$$v = 2\times3.1416\times0.32\times10\times10^6 = 2.01\times10^7\ m/s$$

(ii) From the relation $Bqv = mv^2/r$, we obtain the magnetic field, $B = (mv/qr)$

Here, $v = 2.01\times10^7\ m/s$, m = mass of deutron = $2\times1.67\times10^{-27}kg, r = 0.32m,\ q = 1.60\times10^{-19}C$
Substituting these values in the expression for B, we obtain

$$\therefore \quad B = \frac{2\times1.67\times10^{-27}\times2.01\times10^7}{1.60\times10^{-19}\times0.32} = 1.31\,\text{T}$$

(iii) Energy of deuterons $= mv^2/2$

$$= 2\times1.67\times10^{-27}\times\left(2.01\times10^7\right)^2/2$$

$$= 1.67\times(2.01)^2\times10^{-13}\,J$$

$$= \frac{1.67 \times (2.01)^2 \times 10^{-13}}{1.602 \times 10^{-13}} MeV = \mathbf{4.22\ MHz}$$

4. A cyclotron has a magnetic field of 1.5 Wb/m². The extraction radius is 0.5m. Calculate the frequency of the rf-oscillator necessary for accelerating deutrons and the energy of the extracted beam.

Soln. Frequency, $f = \frac{Bq}{2\pi m} = \frac{1.5 \times 1.6 \times 10^{-19}}{2 \times 3.14 \times 2 \times 1.67 \times 10^{-27}}$

$$= 11.44 \times 10^6\ Hz = \mathbf{11.44\ MHz}$$

Kinetic energy of deuterons, $W_k = \frac{B^2 q^2 r^2}{2m}$. Substituting the given values in the expression for W_k, we get

$$W_k = \frac{(1.5)^2 \times (1.6 \times 10^{-19})^2 \times (0.5)^2}{[illegible]} = 2.155 \times 10^{-12} J$$

$$= \frac{2.155 \times 10^{-12}}{1.6 \times 10^{-13}} MeV = \mathbf{13.46\ MeV}$$

5. What is the energy to which protons can be accelerated in a cyclotron with a dee-dia of 2m and a magnetic field of flux density 0.72 Wb/m². Mass of proton $= 1.673 \times 10^{-27}\ kg$

Soln. From the relation, $\frac{mv^2}{r} = Bqv$, we obtain the expression for the velocity

$$v = \frac{Bqr}{m}$$

Therefore, energy, $W_k = \frac{1}{2} mv^2 = \frac{1}{2m}(Bqr)^2$

$$\therefore \quad W_k = \frac{(0.72 \times 1.6 \times 10^{-19} \times 1)^2}{[illegible]} = 3.966 \times 10^{-12} J$$

$$= \frac{3.966 \times 10^{-12}}{1.6 \times 10^{-13}} MeV = \mathbf{24.78\ MeV}$$

6. An alternating p.d. of 50 kV maximum value is applied to the dees of radius 40 cm of a cyclotron by an oscillator. A deuteron of mass 2 amu acquires energy of 4 MeV in the cyclotron. Calculate (i) magnetic field strength, (ii) frequency of the oscillator and (iii) number of revolutions which the deuteron has to make inside the cyclotron to gain the energy. 1 amu $= 1.6 \times 10^{-27} kg$, $q = 1.6 \times 10^{-19} C$

Soln. Kinetic energy of the emerging beam of particles of mass m, charge q in a cyclotron is given by

$$W_k = \frac{B^2 q^2 r^2}{2m}$$

where, B = the magnetic field and r = the radius of a dee.

(i) $\therefore \quad B^2 = \frac{2W_k m}{q^2 r^2} = \frac{2 \times (4 \times 1.6 \times 10^{-13}) \times (2 \times 1.6 \times 10^{-27})}{(1.6 \times 10^{-19})^2 [illegible]} = 1.0 \quad \therefore \ B = \mathbf{1.0\ Wb/m^2}$

(ii) Resonance frequency, $f = \frac{Bq}{2\pi m} = \frac{1.0 \times 1.6 \times 10^{-19}}{2 \times 3.14 \times (2 \times 1.6 \times 10^{-27})} = 8 \times 10^6\ c/s = \mathbf{8 Mc/s}$

(iii) Let n be the number of revolutions, $n = \frac{W_k}{2qV} = \frac{4 \times 1.6 \times 10^{-13}}{2 \times 1.6 \times 10^{-19} \times 50 \times 10^3} = \mathbf{40}$

Nuclear Reactions

When two nuclei come close together, a nuclear reaction can occur that results in nuclei being formed. Nuclei are positively charged and the repulsion between them keeps them beyond the range where they can interact unless they are moving very fast. In the sun and stars whose internal temperatures range up to millions of Kelvins, many nuclei present have high enough speeds for reaction to be frequent. In lab it is easy to produce nuclear reactions on a small scale, either with α-particles on with protons or heavier nuclei accelerated in various ways.

- **Classification of nuclear reactions:**

(i) **Elastic scattering:** In elastic scattering, the same particles are scattered in different directions and there is no loss of energy. The residual nucleus is the same as the target nucleus and is left in the same state (ground state) as the latter so that it can be represented as X(x, y)X. An example is the scattering of neutrons by graphite: $C(n,n)C$. **Example:** $_2He + _{79}Au \rightarrow _{79}Au + _2He$

(ii) **Inelastic scattering:** In inelastic scattering, the same particles are scattered in different directions with different energy, as there is loss of energy due to collision. The residual nucleus which is the same as target nucleus is left in an excited state so that the process can be represented as $X(x,y)X^*$. An example is the collision of fast neutrons with U-238.

(iii) **Radiative capture:** In radiative capture, the incident particle is absorbed or captured by the target nucleus to form the excited compound nucleus which disintegrates to produce one or more $\gamma-$photons and goes down to the ground state. The process may be represented as $X(x,\gamma)Y^*$. **Example:** $_{92}U^{235} + _0n^1 \longrightarrow _{92}U^{236}$.

(iv) **Reaction of transformation:** Here the oncoming particle is retained in the nucleus and the compound nucleus emits a different particle so that the product nucleus is different from target nucleus, e.g. $^9Be(\alpha,n)^{12}C$

(v) **Photo-disintegration:** In photo-disintegration, a very energetic photon is absorbed by the target nucleus so that it is raised to an excited state and subsequently disintegrates. It can be represented as $X(\gamma,y)Y$. **Example:** $_1H^2 + \gamma \longrightarrow _1H^1 + _0n^1$.

(vi) **Stripping reactions:** In stripping reactions, one or more nucleons from the projectile are captured by the target nucleus, the remaining stripped nucleus is emitted in a different direction.

Example: $_{29}Cu^{63} + _1d^2 \longrightarrow _{29}Cu^{64} + _1H^1$

(vii) **Direct reactions:** A collision of an incident particle with the nucleus may immediately pull one of the nucleons out of the target nucleus and is called 'pick up reaction'.

Example: $_1H^1 + _6C^{13} \longrightarrow _1d^2 + _6C^{12}$

- **Conservation in Nuclear reactions:**

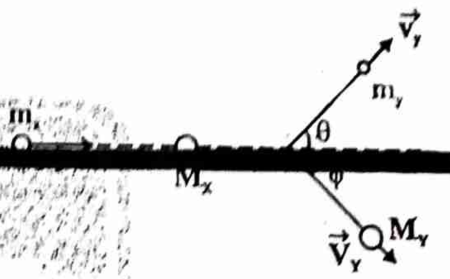
(i) Conservation of mass number
(ii) Conservation of atomic number
(iii) Conservation of energy (including mass-energy)
(iv) Conservation of linear momentum
(v) Conservation of angular momentum.
(vi) Conservation of parity
(vii) Conservation of isotopic spin

Kinematics of nuclear reactions:

Let a particle of mass m_x moving along x-axis with a velocity v_x collide elastically with a target nucleus of mass M_X, at rest. Due to the nuclear reaction, the product nucleus of mass M_Y is scattered at an angle ϕ with velocity v_Y and the product particle of m_y is emitted with velocity v_y at an angle θ

From the law of conservation of linear momentum along x and y directions, we get

$$\left.\begin{aligned} M_Y u_Y \cos\phi + m_y v_y \cos\theta &= m_x v_x \\ -M_Y v_Y \sin\phi + m_y v_y \sin\theta &= 0 \end{aligned}\right\}$$



Or,
$$\left.\begin{aligned} M_Y v_Y \cos\phi &= m_x v_x - m_y v_y \cos\theta \\ M_Y v_Y \sin\phi &= m_y v_y \sin\theta \end{aligned}\right\}$$

Squaring the above two equations and adding, we obtain

$$M_Y^2 v_Y^2 = m_x^2 v_x^2 + m_y^2 v_y^2 - 2m_x m_y v_x v_y \cos\theta$$

Or,
$$2M_Y K_Y = 2m_x K_x + 2m_y K_y - 4\sqrt{m_x m_y K_x K_y}\cos\theta$$

since, non-relativistically, K = kinetic energy $= \frac{1}{2}mv^2$

$$\therefore \quad K_Y = \frac{m_x}{M_Y}K_x + \frac{m_y}{M_Y}K_y - \frac{2}{M_Y}\sqrt{m_x m_y K_x K_y}\cos\theta$$

But, $Q = K_Y + K_y - K_x$

$$\therefore \quad Q = K_y\left(1 + \frac{m_y}{M_Y}\right) - K_x\left(1 - \frac{m_x}{M_Y}\right) - \frac{2}{M_Y}\sqrt{m_x m_y K_x K_y}\cos\theta$$

This gives the Q-value of the reaction in terms of K_y, K_x and θ without involving the kinetic energy of the recoil nucleus, K_Y and the mass M_x of target nucleus and is called the standard form of Q-equation.

If $\theta = 90^\circ$, the Q-expression simplifies to : $\quad Q = K_y\left(1 + \frac{m_y}{M_Y}\right) - K_x\left(1 - \frac{m_x}{M_Y}\right)$

If Q is negative, it gives the minimum value of threshold kinetic energy to be given to the incident particle for an endoergic reaction to proceed in the forward direction.

- **Q-value and threshold energy of nuclear reaction:**

The law of conservation of energy and momentum imposes certain restrictions on the reactions. These restrictions are called the kinematic restrictions and this mathematical methods is known as kinematics.

Consider the nuclear reaction

$$x + X \longrightarrow y + Y$$

Where x, X, y and Y are the bombarding particle, target nucleus, outgoing particle and product nucleus respectively. It is assumed that the target nucleus is in rest. Since total energy is conserved in the nuclear reaction, therefore we get,

$$\left(m_x c^2 + E_x\right) + M_X c^2 = \left(E_y + m_y c^2\right) + \left(E_Y + M_Y c^2\right)$$

E_x, E_y and E_Y are the kinetic energies of respective particles.

Now the quantity $Q = E_y + E_Y - E_x \Rightarrow Q = \left(m_x + M_X - m_y - M_Y\right)c^2$

Where Q is called the Q-value of nuclear reaction.

(i) If Q is positive, the reaction is said to be exoergic (exothermic) and

(ii) If Q is negative, the reaction is called endoergic (endothermic).

The minimum K.E. required for incident particle (x) to start the nuclear reaction is called the threshold energy $(E_x)^{th}$. The relation between Q-values and threshold energy is:

$$E_x^{th} = -Q\frac{\left(m_y + M_Y\right)}{\left(M_Y + m_y - m_x\right)}$$

If $E_x^{th} = 0$ for exoergic or exothermic reactions i,e. these reaction are spontaneous process.

- **Mechanism of nuclear reactions:**
- **Transmutation by protons:**

(a) (p, α) reactions:

(i) ${}_3Li^7 + {}_1H^1 \rightarrow \left({}_4Be^8\right)^* \rightarrow {}_2He^4 + {}_2He^4$

(ii) ${}_3Li^6 + {}_1H^1 \rightarrow \left({}_4Be^7\right)^* \rightarrow {}_2He^3 + {}_2He^4$

(iii) ${}_4Be^9 + {}_1H^1 \rightarrow \left({}_5B^{10}\right)^* \rightarrow {}_3Li^6 + {}_2He^4$

(iv) ${}_5B^{11} + {}_1H^1 \rightarrow \left({}_6C^{12}\right)^* \rightarrow {}_4Be^8 + {}_2He^4$

(v) ${}_9F^{19} + {}_1H^1 \rightarrow \left({}_{10}Ne^{20}\right)^* \rightarrow {}_8O^{16} + {}_2He^4$

(b) (p–n) Reaction:

(i) ${}_5B^{11} + {}_1H^1 \rightarrow \left({}_6C^{12}\right)^* \rightarrow {}_6C^{11} + {}_0n^1$

(ii) ${}_8O^{18} + {}_1H^1 \rightarrow \left({}_9F^{19}\right)^* \rightarrow {}_9F^{18} + {}_0n^1$

(iii) ${}_{11}Na^{23} + {}_1H^1 \rightarrow \left({}_{12}Mg^{24}\right)^* \rightarrow {}_{12}Mg^{23} + {}_0n^1$

(iv) ${}_{28}Ni^{58} + {}_1H^1 \rightarrow \left({}_{29}Cu^{59}\right)^* \rightarrow {}_{29}Cu^{58} + {}_0n^1$

(v) ${}_{29}Cu^{65} + {}_1H^1 \rightarrow \left({}_{30}Zn^{66}\right)^* \rightarrow {}_{30}Zn^{65} + {}_0n^1$ (usually endoergic)

(c) (p, d) reaction:

(i) ${}_3Li^7 + {}_1H^1 \rightarrow {}_3Li^6 + {}_1H^2$

(ii) ${}_4Be^9 + {}_1H^1 \rightarrow {}_4Be^8 + {}_1H^2$

(d) Proton capture:

Compound nucleus in excited states come to ground state with γ-ray photon.

(i) ${}_3Li^7 + {}_1H^1 \rightarrow \left({}_4Be^8\right)^* \rightarrow {}_4Be^8 + \gamma$

(ii) ${}_6C^{12} + {}_1H^1 \rightarrow \left({}_7N^{13}\right)^* \rightarrow {}_7N^{13} + \gamma$

(iii) ${}_7N^{14} + {}_1H^1 \rightarrow \left({}_8O^{15}\right)^* \rightarrow {}_8O^{15} + \gamma$

(iv) ${}_9F^{19} + {}_1H^1 \rightarrow \left({}_{10}Ne^{20}\right)^* \rightarrow {}_{10}Ne^{20} + \gamma$

(v) $_{13}Al^{27} + _1H^1 \rightarrow (_{14}Si^{28})^* \rightarrow _{14}Si^{28} + \gamma$ (vi) $_{24}Cr^{50} + _1H^1 \rightarrow (_{25}Mn^{51})^* \rightarrow _{25}Mn^{51} + \gamma$

If incident proton has an energy (> 20 MeV) the compound nucleus has sufficient excitation energy to permit the expulsion of two or more nucleons.

[redacted]

Neutrons have no electric charge and can penetrate +vely charged nuclei without any experience of repulsive electrostatic force.

(a) (n–α) reaction:
With slow Neutrons

(i) $_3Li^6 + _0n^1 \rightarrow (_3Li^7)^* \rightarrow _1H^3 + _2He^4$ (ii) $_5B^{10} + _0n^1 \rightarrow (_5B^{11})^* \rightarrow _3Li^7 + _2He^4$

(iii) $_{13}Al^{27} + _0n^1 \rightarrow (_{13}Al^{28})^* \rightarrow _{11}Na^{24} + _2He^4$ (iv) $_7N^{14} + _0n^1 \rightarrow (_7N^{15})$

(v) $_5B^{11} + _2He^4$ (vi) $_3Li^7 + _2He^4 + _2He^4$

Capture of fast Neutrons - emission of α - particle are usually radioactive

(i) $_{11}Na^{23} + _0n^1 \rightarrow (_{11}Na^{24})^* \rightarrow _9F^{20} + _2He^4$ followed by $_9F^{20} \rightarrow _{10}Ne^{20} + \beta^-$

(ii) $_{13}Al^{27} + _0n^1 \rightarrow (_{13}Al^{28})^* \rightarrow _{11}Na^{24} + _2He^4$ followed by $_{11}Na^{24} \rightarrow _{12}Mg^{24} + \beta^-$

(b) (n, p) reaction: Proton in the nucleus is replaced by neutron mass no, does not change but change decreases by one unit.

(i) $_7N^{14} + _0n^1 \rightarrow (_7N^{15})^* \rightarrow _6C^{14} + _1H^1 + Q$ followed by $_6C^{14} \rightarrow _7N^{14} + \beta^- + E_{max}$

(ii) $_2He^3 + _0n^1 \rightarrow (_2He^4)^* \rightarrow _1H^3 + _1H^1 + Q$ followed by $_1H^3 \rightarrow _2He^3 + \beta^- + E_{max}$

(iii) $_{17}Cl^{35} + _0n^1 \rightarrow (_{17}Cl^{36})^* \rightarrow _{16}S^{35} + _1H^1 + Q$ followed by $_{16}S^{35} \rightarrow _{17}Cl^{35} + \beta^- + E_{max}$

With fast neutrons:

(i) $_{13}Al^{27} + _0n^1 \rightarrow (_{13}Al^{28})^* \rightarrow _{12}Mg^{27} + _1H^1$ **(ii)** $_{16}S^{32} + _0n^1 \rightarrow (_{16}S^{33})^* \rightarrow _{15}P^{32} + _1H^1$

(iii) $_{29}Cu$ [redacted] $+ _0n$ [redacted] $\rightarrow (_{29}Cu$ [redacted] $) \rightarrow _{28}Ni$ [redacted] $+ _1H$ [redacted] (iv) $_{30}Zn$ [redacted] $+ _0n$ [redacted] $\rightarrow (_{30}Zn$ [redacted] $) \rightarrow _{29}Cu$ [redacted] $+ _1H$ [redacted]

(c) $(n-\gamma)$ Reaction:

(i) $_1H^1 + _0n^1 \rightarrow (_1H^2)^* \rightarrow _1H^2 + \gamma$ **(ii)** $_1H^2 + _0n^1 \rightarrow (_1H^3$

Product Nucleus is Radioactive:

(i) $_{45}Rh^{103} + _0n^1 \rightarrow (_{45}Rh^{104})^* \rightarrow _{45}Rh^{104} + \gamma$ **(ii)** $_{49}In^{115} + _0n^1 \rightarrow ($

(iii) $_{92}U^{238} + _0n^1 \rightarrow (_{92}U^{239})^* \rightarrow _{92}U^{239} + \gamma$ **(iv)** $_{29}Cu^{65} + _0n^1$

(v) $_{79}Au^{65} + _0n^1 \rightarrow (_{79}Au^{198})^* \rightarrow _{79}Au^{198} + \gamma$

(d) (n – d) and (n – t) reaction: Bombardment of 90 MeV neutrons: ${}_7N^{14} + {}_0n^1 \rightarrow {}_6C^{12} + {}_1H^3$

(e) (n – 2n) reaction: one neutron captured by nucleus and 2 neutrons are emitted. Q < 0 fast neutron are needed . Most cases residue nucleus unstable -followed by positron emission.

(i) ${}_6C^{11} + {}_0n^1 \rightarrow \left({}_6C^{13}\right)^* \rightarrow {}_6C^{11} + 2\,{}_0n^1$ (ii) ${}_{19}K^{39} + {}_0n^1 \rightarrow \left({}_{19}K^{40}\right)^* \rightarrow {}_{19}K^{38} + 2\,{}_0n^1$

(iii) ${}_{51}Sb^{121} + {}_0n^1 \rightarrow \left({}_{51}Sb^{122}\right)^* \rightarrow {}_{51}Sb^{120} + 2\,{}_0n^1$

(f) Neutron - Three or more particles: Incident Neutron ~ 30 MeV sufficient energy to overcome coulomb Barrier, 3 neutrons or even 2 neutron and a proton are ejected from compound nucleus. Neutron (~ 100 MeV) Nuclei with moderate mass no. undergo spallation and those of high mass no. eg. Bi and Pb suffer fission probably accompanied by spallative.

- **(3) Transmutation by Deuteron: High energy Deuteron.**

(a) $(d-\alpha)$ reaction

(i) ${}_3Li^6 + {}_1H^2 \rightarrow \left({}_4Be^8\right)^* \rightarrow {}_2He^4 + {}_2He^4$, (ii) ${}_8O^{16} + {}_1H^2 \rightarrow \left({}_9F^{18}\right)^* \rightarrow {}_7N^{14} + {}_2He^4$,

(iii) ${}_{10}Ne^{20} + {}_1H^2 \rightarrow \left({}_{11}Na^{22}\right)^* \rightarrow {}_9F^{18} + {}_2He^4$

(iv) ${}_{12}Mg^{26} + {}_1H^2 \rightarrow \left({}_{13}Al^{28}\right)^* \rightarrow {}_{11}Na^{24} + {}_2He^4$,

(v) ${}_{13}Al^{27} + {}_1H^2 \rightarrow \left({}_{14}Si^{29}\right)^* \rightarrow {}_{12}Mg^{25} + {}_2He^4$

(b) (d – p) Reaction: Isotope creation

(i) ${}_6C^{12} + {}_1H^2 \rightarrow \left({}_7N^{14}\right)^* \rightarrow {}_6C^{13} + {}_1H^1$ (ii) ${}_3Li^7 + {}_1H^2 \rightarrow \left({}_4Be^9\right)^* \rightarrow {}_3Li^8 + {}_1H^1$

(iii) ${}_{11}Na^{23} + {}_1H^2 \rightarrow \left({}_{12}Mg^{25}\right)^* \rightarrow {}_{11}Na^{24} + {}_1H^1$ (iv) ${}_{15}P^{31} + {}_1H^2 \rightarrow \left({}_{16}S^{33}\right)^* \rightarrow {}_{15}P^{32} + {}_1H^1$

(v) ${}_{48}Cd^{114} + {}_1H^2 \rightarrow \left({}_{49}In^{116}\right)^* \rightarrow {}_{48}Cd^{115} + {}_1H^1$

(vi) ${}_{83}Bi^{209} + {}_1H^2 \rightarrow \left({}_{84}Po^{211}\right)^* \rightarrow {}_{83}Bi^{210} + {}_1H^1$

(c) (d – n) Reaction:

(i) ${}_3Li^7 + {}_1H^2 \rightarrow \left({}_4Be^9\right)^* \rightarrow {}_4Be^8 + {}_0n^1$ (ii) ${}_4Be^9 + {}_1H^2 \rightarrow \left({}_5B^{11}\right)^* \rightarrow {}_5B^{10} + {}_0n^1$

(iii) ${}_6C^{12} + {}_1H^2 \rightarrow \left({}_7N^{14}\right)^* \rightarrow {}_7N^{13} + {}_0n^1$

When two Deuterons Interact both the (d, n) and (d, p) reactions have been obtained.

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4 \rightarrow {}_1H^3 + {}_1H^1 + 4.02\text{ MeV}$$

$$\rightarrow {}_2He^3 + {}_0n^1 + 3.25\text{ MeV}$$

- **Nuclear fission and fusion :**

Fission and fusion are two processes that alter the nucleus of an atom. Nuclear fission provides the energy in nuclear power plants and fusion is the source of the sun's energy. The use of fission in power plant can help conserve fossil fuels. Without the energy produced by the fusion of hydrogen in the sun, the earth would quickly change into a cold planet that could not support life as we know it.

Nuclear transformation always obey two fundamental conservation laws

(i) Mass number is conserved

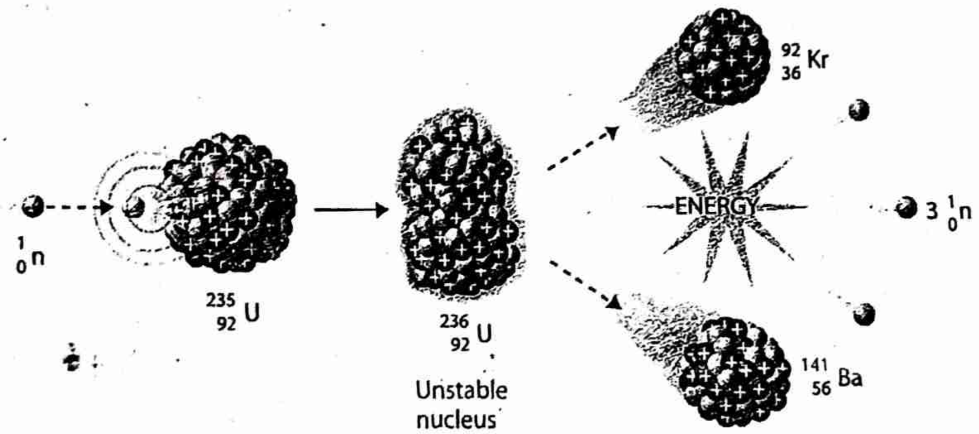
(ii) Electric charge is conserved.

Energy and mass are not conserved but can be interconverted according to Einstein's equation.

$$E = MC^2$$

- **Nuclear fission:**

The process of fission occure when a nucleus spits into smaller pieces. Fission can be induced by a nucleus capturing slow moving neutrons, which result in the nucleus becoming very unstable.



The following equation represent fission reactions

$$^{235}U_{92} + {}^{1}n_{0} \longrightarrow {}^{141}Ba_{56} + {}^{92}Kr_{36} + 3{}^{1}n_{0}$$

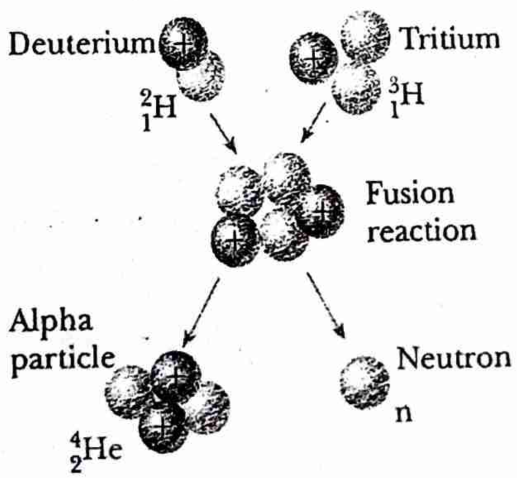
$$^{235}U_{92} + {}^{1}n_{0} \longrightarrow {}^{137}Te_{52} + {}^{97}Zr_{40} + 2{}^{1}n_{0}$$

$$^{235}U_{92} + {}^{1}n_{0} \longrightarrow {}^{152}Nd_{66} + {}^{81}Ge_{32} + 3{}^{1}n_{0}$$

$$^{235}U_{92} + {}^{1}n_{6} \longrightarrow {}^{131}Sn_{50} + {}^{103}Mo_{42} + 2{}^{1}n_{0}$$

- **Fusion :**

Fusion occurs when two nuclei joint together to form a large nucleus. Fusion is brought about by bringing together two or more small nuclei under conditions of tremendous pressure and heat.



The following equation represent fusion reactions

$$^2H_1 + {}^2H_1 \longrightarrow {}^3H_1 + {}^1p_1$$

$$^2H_1 + {}^3H_1 \longrightarrow {}^4He_2 + {}^1n_0$$

$$^3He_2 + {}^3He_2 \longrightarrow {}^4He_2 + 2{}^1H_1$$

- **Condition of fusion reaction of heat :**

Thermal kinetic energy = Coulomb's repulsion

$$\Rightarrow \quad \boxed{\frac{3}{2}kT = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}}$$

- **Cross-section of nuclear reactions (Classical):**

One of the most important parameters in nuclear reactions is the reaction cross-section, symbolised by σ. It is quantitative measure of the probability of occurrence of a nuclear reaction.

Let a parallel beam of N monoenergetic particles be incident per unit time normally on a target foil of surface area A and thickness Δx, having 'n' nuclei per unit volume. Now, the number ΔN of nuclei in the foil undergoing nuclear reaction will be proportional to (i) the intensity of the beam and (ii) the number of target nuclei contained in the the foil.

But the intensity I of the beam = N/A and the number of nuclei in the foil = $nA.\Delta x$

Therefore, number of nuclei transmuted is $\Delta N \propto \frac{N}{A} nA.\Delta x$

$\therefore \quad \Delta N = \sigma Nn.\Delta x = \sigma Nn_1$ where σ = constant.

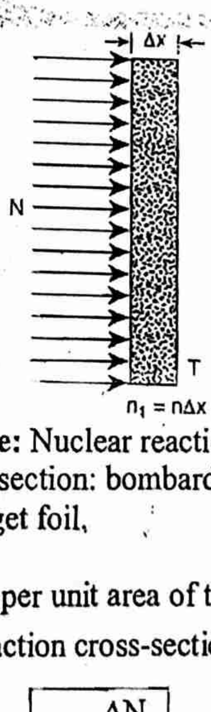


Figure: Nuclear reaction cross-section: bombardment of target foil,

where $n_1 = n.\Delta x$, the number of nuclei per unit area of the target foil used.

The constant σ is called the nuclear reaction cross-section.

Therefore, nuclear reaction cross-section, $\boxed{\sigma = \frac{\Delta N}{Nn_1}}$

- The probability of the incident particle to interact with the target atom will be equal to

$$P = \frac{\text{number of interact particle}}{\text{number of incident particle}}$$

If dN is number of interacting particle and N is number of incident particles, then

$$P = \boxed{\frac{dN}{N} = \sigma\, n\, dx} \Rightarrow \int_{N_0}^{N} \frac{dN}{N} = \int_{z=z}^{z=0} \sigma\, n\, dx \Rightarrow \ell n\left[\frac{N}{N_0}\right] = -\sigma\, n\,(+x) \Rightarrow \frac{N}{N_0} = e^{-\sigma n x}$$

$$\boxed{N = N_0\, e^{-\sigma n x}}$$

● **Neutron - Proton Scattering at low energies**

- Nucleus is a bound system means attractive force exists b/w neutron and proton.
  In scattering of free neutrons by protons a parallel beam of Neutron is allowed to impinge upon a target containing hydrogen atoms and no. of Neutrons dejected through various angles is determined as a function of Neutron energy.
- Since neutron have no charge, they are unaffected by the electrostatic field and their scattering will directly reflect the operation of nuclear force.

Two kinds of reactions can be involved in Neutron-proton interaction
(i) Scattering
(ii) Radiative Capture: It has low probability for high energy neutron as the cross-section for the competing radiative capture reaction decreases with $\frac{1}{2} \rightarrow$ neutron velocity

In practice protons are bound in nucleus the chemical binding energy of the proton in a molecule is about 0.1eV.
Thus for neutron energy > .1 eV proton can be assumed free. This sets lower limit the neutron energy.
If the neutron energy is less than 10 MeV, only the S-wave overlaps with the Nuclear potential and is scattered.
In the centre of mass system, the Schrodinger equation for the two body (n –p) system is

$$\nabla^2\Psi + \frac{M}{\hbar^2}[E - V(r)]\Psi = 0$$

Where M = proton or Neutron mass = 2 × reduced mass of the system.
E = Incident K.E. in cm system = $\frac{1}{2}$ (Incident K.E in L-coordinate) and V(r) = Inter Nucleon potential energy.
At large distances from the centre of scattering the soln of this equation is expected to be of the form

$$\psi = \phi_{inc}(\bar{r}) + \phi_{sc}(\bar{r}) \qquad \phi_{inc} = A\, e^{i(\bar{k}_0 \cdot \bar{r})} \text{ incident wave function}$$

$$\phi_{sc} = \text{scattering wave function}$$

$$= A\, e^{i(\bar{k}_0 \cdot \bar{r})} + A f(\theta) \frac{e^{i(\bar{k} \cdot \bar{r})}}{r}$$

where, $f(\theta)$ is the scattering amplitude in a direction θ and the differential crosssection is given by

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

If we assume that the potential is spherically symmetric and scattering is ellastic scattering, then amplitude is given by

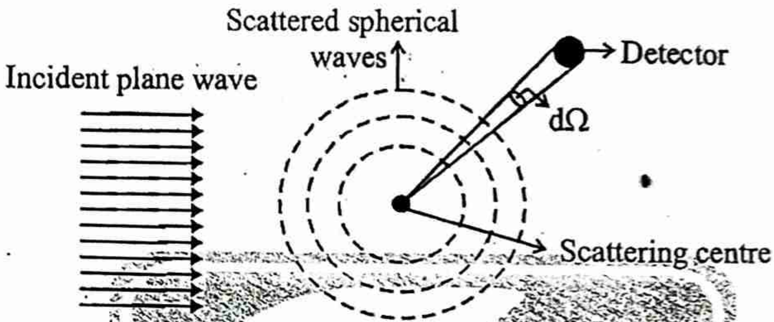
$$\boxed{f(\theta) = \frac{1}{k} \sum_{\ell=0}^{\infty} (2\ell+1) e^{i\delta_\ell} \sin\delta_\ell\, p_\ell(\cos\theta)}$$

Total crosssection is given by

$$\sigma = \iint |f(\theta)|^2 \, d\Omega$$

$\Rightarrow$ $$\sigma = 2\pi \iint |f(\theta)|^2 \sin\theta d\theta$$

$\Rightarrow$ $$\boxed{\sigma = \frac{4\pi}{k^2}\sum(2\ell+1)\sin^2\delta_\ell}$$



Inelastic scattering:

It $\eta_\ell(k)$ is the loss factor and $0 \le \eta_\ell(k) \le 1$ for enelastic scattering, scattering amplitude

$$f(\theta) = \frac{1}{2k}\sum_{\ell=0}^{\infty}(2\ell+1)\left[\eta_\ell \sin 2\delta_\ell + i(1-\eta_\ell \cos\delta_\ell)\right]P_\ell(\cos\theta)$$

Total elastic scattering cross section is given by

$$\boxed{\sigma_{el} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left(1+\eta_\ell^2 - 2\eta_\ell \cos 2\delta_\ell\right)}$$

Total inelastic scattering cross section is given by

$$\boxed{\sigma_{inel} = \frac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left(1-\eta_\ell^2(k)\right)}$$

And the total cross-section is given by

$$\boxed{\sigma_{tot} = \frac{2\pi}{k^2}\sum(2\ell+1)\left(1-\eta_\ell \cos(2\delta_\ell)\right)}$$

* $\eta_\ell = 0$ pure inelastic scattering.

  $\eta_\ell = 1$ for pure elastic scattering.

- **Scattering length:**
- For neutrons of very low energy scattered by free protons $\lambda\!\!\!^{-}$ is very large and hence k is very small we have $f(\theta) = \frac{e^{i\delta_0}}{k}\sin\delta_0$ and scattering cross-section $\sigma_{sc} = 4\pi\lambda\!\!\!^{-2}\sin^2\delta_0$

as $k \to 0$ δ_0 must approach zero or $f(\theta)$ would become infinite.

- For low energy neutrons $f(\theta)$ can be written as $f(\theta) \lim_{\delta_0 \to 0} \frac{e^{i\delta_0} \sin\delta_0}{k} = \frac{\delta_0}{k} = -a$ where quantity +a is called scattering length.
- The zero energy scattering cross-section is $\sigma_0 = 4\pi a^2$ (Identical to impenetrable sphere of radius a).

- **Deuteron:**

Deuteron is the only two-nucleon bound system made up of a proton and a neutron. The two other possible two nucleon system the Diproton and Dineutron do not exist as bound system.

- **Experimentally Determined Properties of Deuteron**

(i) The BE of Deuteron is very small . Its value is 2.225 ± 0.003 MeV. This is very less as compared to the stable nuclei. i.e, 8MeV, hence loosely bound.

(ii) The Angular momentum quantum no . often called the nuclear spin of ground state of the Deuteron is 1. It suggest that spins are parallel (triplet state) and the orbital angular momentum of the Deuteron about their common center of mass is zero. This the ground state is ^3S state.

(iii) The sum of the magnetic dipole moments of the proton (2.79275 μ_N) and Neutron (–1.91315 μ_N), do not exactly equal to magnetic moment of Deuteron (0.85735 μ_N)

(iv) Deuteron has small but +ve Quadrupole moment suggesting that it deviates from spherical shape and has the probability of finding in the next higher state i.e., ^3D state also. Hence and state is a mixture of ^3S and ^3D state.

(v) n-p Combination, neutron is uncharged hence the force is not electrostatic, mass is less hence no gravitational , must be of nuclear origin and assumed to be central and attractive. This assumption somewhat disagrees with the experiment (central).

- Consider the example $\mu_p = 2.79281 \pm 0.00004$ nm and $\mu_n = -1.913148 \pm 0.000066$ nm

  The fact that $\mu_d + \mu_n - \mu_d \neq 0$ although small. suggests the Deuteron may not be fully described by the spherically symmetric $^3s_1$ state.

- Moreover is nuclear force if due to the exchange of Mesons, the magnetic moments of the Nucleons where in the free state may not be same as when in the nucleus. Correction to magnetic moment due to Mesonic current.
- D-state contribution can be considered by Non-Central Tensor free.

  By Assuming Central potential.

Schrodinger equation $-\frac{\hbar^2}{2\mu}\nabla^2\psi(r) + v(r)\psi(r) = E\psi(r)$

where reduced mass $\mu = \frac{m_p m_n}{m_p + m_n} = M/2$

Now equation can be written as $\frac{\hbar^2}{M}\frac{d^2u(r)}{dr^2} + v(r)u(r) = Eu(r)$

where $\psi(r) = \frac{u(r)}{r}$ and E = – W = –2.226 MeV = binding energy of Deuteron

- **Various types of potential**

Square well potential $V(r) = -V_0 \quad r \le r_0$

$= 0 \quad r > r_0$

Exponential $V(r) = -V_0 e^{-r/r_0}$, Gaussion, $V(r) = -V_0 e^{-r^2/r_0^2}$

Yukawa $V(r) = \dfrac{-V_0 e^{-r/r_0}}{r/r_0}$

- **Ground state of deutron ($_1D^2$)**

Deutron is a single two nucleon (one P and One n) bound system which is found in nature. The important experimental determinal properties about deutron are given below:

(i) The B.E. of deutron/nucleon is very small compare to other nuclei i.e. it is a weakly system.

(ii) The ground state spin of deutron $I_d = 1$ (iii) The ground state parity of deutron = even (+)

(iv) The quadrupole momento of deutron $(Q_d) \neq 0$

(v) The magnetic moment of deutron is slightly different from the sum of intrinsic mag. moments of neutron + proton i.e. $(\mu_n + \mu_p) - \mu_d \to 0$

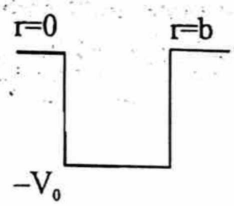
These factor represents that the ground state of Deutron is a mixture of $\underset{(L=0)}{^3S}$ & $\underset{(L=2)}{^3D}$ states in which ^3S contribution is 96% and ^3D contribution is only 4%.

This indicate that nuclear force are non-central as $L \neq$ constant and they are spin dependent. In deutron both p and n have spin parallel to each other.

- **Schrodinger wave equation for Deutron and its solution for central force (L = 0):**

As deutron is a bound system, let us consider the case of rectangular pot. well represented as

$$\begin{bmatrix} V = -V_0 & 0 < r < b \\ = 0 & \text{elsewhere} \end{bmatrix}$$



The schrodinger equation is $\boxed{\nabla^2\psi + \dfrac{2\mu}{\hbar^2}(E - V)\psi = 0}$

$$\psi = \psi(r,\theta,\phi) = \sum_\ell R_\ell(r) Y_{\ell m}(\theta,\phi) = \sum \frac{u_\ell(r)}{r} Y_{\ell m}(\theta,\phi)$$

where reduce mass $\mu = \dfrac{m_n m_p}{m_n + m_p} \Rightarrow \mu \sim M/2 \,(m_n \sim m_p = M)$

For $\ell = 0$. We get, $\boxed{k_1 \cot k_1 b = -\alpha}$

or
$$\cot k_1 b = -\frac{\alpha}{k_1} = -\frac{\sqrt{\dfrac{ME_d}{\hbar^2}}}{\sqrt{\dfrac{M}{\hbar^2}(V_0 - E_d)}} = -\sqrt{\frac{E_d}{V_0 - E_d}}$$

as E_d is very small, then an approximate solution of equation is obtained as $\cot\sqrt{\frac{M}{\hbar^2}(V_0 - E_d)}.b \rightarrow 0$

$$\Rightarrow \quad \cot\sqrt{\frac{M}{\hbar^2}V_0}.b = \cot\pi/2, \cot 3\pi/2 ......$$

A minimum value of V_0 can be obtained by setting $b\sqrt{\frac{M}{\hbar^2}V_{0m}} = \pi/2$

$$\Rightarrow \quad V_{0m} = \frac{\pi^2\hbar^2}{4Mb^2} \Rightarrow \boxed{V_{0m}b^2 =} \text{ constant}$$

$V_0 \sim 25$ MeV is obtained by putting b is 2×10^{-15} m

The exact solution of equation (9) is obtained by graphical method in which $V_{0m} \sim 38$ MeV

Solved Examples

1. The threshold temperature above which the thermonuclear reaction: **[GATE 2005]**

$${}_2^3He + {}_2^3He \rightarrow {}_2^4He + 2{}_1^1H + 12.86\,MeV$$

can occur is (use $e^2/4\pi\varepsilon_0 = 1.44\times10^{-15}$ MeVm)

(a) 1.28×10^{10} K (b) 1.28×10^{9} K (c) 1.28×10^{8} K (d) 1.28×10^{7} K

Soln. If T be the threshold temperature of the reaction then,

$$\frac{3}{2}kT = \frac{4e^2}{4\pi\varepsilon_0 r}$$

$$\Rightarrow T = \frac{2}{3} = \frac{4\times1.44\times10^{-15}\times1.6\times10^{-13}}{1.38\times10^{-23}\times2.4\times1.44\times10^{-5}} = 1.28\times10^{10}\,K$$

$$\left[\begin{array}{l} \frac{e^2}{4\pi\varepsilon_0} = 1.44\times10^{-15}\times1.6\times10^{-13}\,Jm \\ r = 2r_0A^{1/3} = 2\times1.2\times10^{-15}(3)^{1/3}\,m \\ k = 1.38\times10^{-23}\,kgm^2\sec^{-2}K^{-1} \end{array}\right]$$

Correct option is (a)

2. The energy released in the fission of 1 kg uranium (approximately [in Joule]): **[GATE 2008]**

(a) 10^{14} (b) 10^{17} (c) 10^{16} (d) 10^{10}

Soln. We know that binding energy per nucleon of uranium is 7.6 MeV and binding energy per nucleo of daughter nucleon is 8.5

Therefore, energy released from 1 kg uranium

$$= (8.5 - 7.6)\times235\times\frac{6.023\times10^{23}}{235}\times10^3\times1.6\times10^{-13} \simeq 10^{14}\;Joule$$

Correct option is (a).

3. In a nuclear reactor, Plutonium $\left({}_{94}^{239}Pu\right)$ is used as fuel, releasing energy by its fission into isotopes of Barium $\left({}_{54}^{146}Ba\right)$ and Strontium $\left({}_{38}^{91}Sr\right)$ through the reaction **[TIFR 2011]**

$${}_{49}^{239}Pu + {}_0^1n \rightarrow {}_{56}^{146}Ba + {}_{38}^{91}Sr + 3\times{}_0^1n$$

... nuclides is given in the table below :

Nuclide	$^{239}_{94}Pu$	$^{146}_{54}Ba$	$^{91}_{38}Sr$
B.E. per nucleon (MeV)	7.6	8.2	8.6

Using this information, One can estimate the number of such fission reactions per second in a 100 MW reactor as

(a) 3.9×10^{18} (b) 7.8×10^{18} (c) 5.2×10^{19} (d) 5.2×10^{18} (e) 8.9×10^{17}

Soln. Energy released from one platonium is

$$=(8.2\times1468.6\times91-7.6\times239)\,MeV$$

$$=1197.2+782.6-1816.4$$

$$=163.4\,MeV$$

$$=163.4\times1.6\times10^{-13}\ joule$$

Let N be number of fission that to be place within second. Then

$$N\times163.4\times1.6\times10^{-13}=100\times10^{6}\ watt$$

$$\Rightarrow\quad N=\frac{100\times10^{6}}{163.4\times1.6\times10^{-13}}=3.8\times10^{18}\simeq3.9\times10^{18}$$

Correct option is (a)

4. Consider the process $\mu^{+}+\mu^{-}\rightarrow\pi^{+}+\pi^{-}$. The minimum kinetic energy of the muons (μ) in the centre of mass frame required to produce the pion (π) pairs at rest is ________ MeV. **[GATE 2014]**

(Given: $m_{\mu}=105\ MeV/c^{2}, m_{\pi}=140\ MeV/c^{2}$)

Soln. According to energy conservation rule

(K.E. + restmass energy)parent = (K.E. + restmass energy) daughter

$$\Rightarrow\quad (K.E.)_{parent}\times2=0+(M_{\pi}-M_{\mu})\times2$$

$$\Rightarrow\quad (K.E.)_{parent}=(140-105)=35\ MeV/c^{2}$$

5. The atomic masses of $^{152}_{63}Eu$, $^{152}_{62}Sm$, $^{1}_{1}H$ and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (*amu*), respectively. Using the above information, the *Q*-value of the reaction $^{152}_{63}Eu+n\rightarrow{}^{152}_{62}Sm+p$ is________ $\times10^{-3}$ *amu* (upto three decimal places) **[GATE 2015]**

Soln. $Q=(M_{Eu}+M_{n})-(M_{sm}+M_{p})$

$$=(151.921749+1.008665)-(151.919756+1.007825)=2.833\times10^{-3}\ a.m.u.$$

Correct answer is (2.833)

6. Calculate the energy generated in MeV when 0.1 kg of ^{7}Li is converted to ^{4}He by proton bombardment.
Given: masses of ^{7}Li, ^{4}He and ^{1}H in a.m.u. are 7.0183, 4.0040 and 1.0081 respectively.

Soln. $^{7}_{3}Li+{}^{1}_{1}H=2\,{}^{4}_{2}He+Q$

Masses of the reactants = (7.0183 + 1.0081) a.m.u. = 8.0264 a.m.u.

Masses of the products = (2×4.0040) a.m.u. = 8.0080 a.m.u.

Therefore, Difference = 0.0184 a.m.u.

therefore, the amount of energy liberated is ... kg. For 0.1 kg of Li,

$$E = \frac{0.1\times0.184}{7.0183}\text{kg} = \frac{0.1\times0.0184}{7.0183}\times\left(3\times10^8\right)^2 \text{J} = \frac{0.1\times0.0184\times9\times10^{16}}{7.0183\times1.6\times10^{-13}}\text{MeV} = 14.74\times10^{25}\text{ MeV}$$

7. Calculate the binding energy in MeV of ^4He from the following data: Mass of ^4He = 4.003875 a.m.u; mass of ^1H = 1.008145 a.m.u. and mass of a neutron = 1.008986 a.m.u.

Soln. A ^4He-nucleus consists of 2 protons and 2 neutrons.

Mass of (2 protons + 2 neutrons) = 2(1.008145 + 1.008986) a.m.u.
= 4.034262 a.m.u.

Mass of ^4He-nucleus = 4.003875 a.m.u.

Therefore, Mass difference = 0.030387 a.m.u.

Binding energy = 0.030387 a.m.u. × c² = (931×0.030387) a.m.u. = 28.29 MeV

8. Calculate the threshold energy for the nuclear reaction $^{14}N(n,\alpha)^{11}B$ in MeV

Soln. Masses of reactants = (14.007550 + 1.008987) a.m.u. = 15.016537 a.m.u.
Masses of products = (4.003879 + 11.012811) a.m.u. = 15.016690 a.m.u.
Therefore,

$$Q = (15.016537 - 15.016690)\text{a.m.u.} = -0.000153\text{ a.m.u.} = -0.000153\times931\text{ MeV} = -0.14\text{MeV}$$

$$\Rightarrow E_{th} = -Q\left[1+\frac{M(n)}{M(N)}\right] = 0.14\left(1+\frac{1.008987}{14.007550}\right)\text{MeV} = 0.14\left(1+\frac{1}{14}\right)\text{MeV} = 0.15\text{ MeV}$$

9. Find the amount of energy in joule released during the process in which 0.001 kg of radium is converted into lead (masses: ^{226}Ra = 226.0955 a.m.u., ^{206}Pb = 206.0386 a.m.u. and α – particle = 4.003 a.m.u.)

Soln. In the conversion of 1 atom of ^{226}Ra into 1 atom of ^{206}Pb, 5α – particles are emitted in all.

Initial mass of ^{226}Ra = 226.0955 a.m.u.
Final mass of ^{206}Pb = 206.0386 a.m.u.

Therefore, Difference in masses = 20.0569 a.m.u.
Mass of 5α – particles = 20.0150 a.m.u.

Therefore, Mass converted into energy = 0.0419 a.m.u.

This is equivalent to an energy (0.0419 ×931) a.m.u. or 39 MeV (for 1 atom)

Now, 0.001 kg (= 1 g) of radium contains $\frac{6.023\times10^{23}}{226}$ Ra – atoms

$$\text{Therefore, Total energy released, } E = \frac{6.023\times10^{23}\times39}{226}\text{MeV} = \frac{6.023\times10^{23}\times39\times1.6\times10^{-13}}{226}\text{J}$$

$$= 16.63\times10^9\text{ J}$$

10. The Q-values in MeV of the following three reactions

$${}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H;\ {}^2_1H + {}^2_1H \rightarrow {}^3_2H + {}^1_0n \text{ and } {}^3_1H \rightarrow {}^2_3He + \beta^-$$

are 4.031, 3.265 and 0.0185 a.m.u. respectively. Calculate the mass difference between the neutron and the hydrogen atom from these data.

Soln. We have: $Q_1 = 4.031\text{ MeV} = \left(\text{mass of } 2{}_1^2\text{H} - \text{mass of } {}_1^3\text{H} - \text{mass of } {}_1^1\text{H}\right)\times c^2$

$$Q_2 = 3.265\text{ MeV} = \left(\text{mass of } 2{}_1^2\text{H} - \text{mass of } {}_2^3\text{H} - \text{mass of } {}_0^1\text{n}\right)\times c^2$$

$$Q_3 = 0.0185\text{ MeV} = \left(\text{mass of } {}_1^3\text{H} - \text{mass of } {}_2^3\text{He}\right)\times c^2$$

The mass of β^- is too small to be taken into account and has been neglected

$\Rightarrow$ $$Q_1 - Q_2 + Q_3 = \left(\text{mass of } {}_0^1\text{n} - \text{mass of } {}_1^1\text{H}\right)\times c^2$$

Therefore, Mass difference between neutron and H-atom is thus

$$Q_1 - Q_2 + Q_3 = (4.031 - 3.265 + 0.0185)\times 931.5\text{ MeV} = 0.7845\text{ MeV}$$

Mass difference = 0.7845 / 931 a.m.u. = 0.000842 a.m.u.

11. Calculate the Q-values of the following two reactions

(i) ${}_1^3\text{H} + {}_1^2\text{H} \rightarrow {}_2^4\text{He} + {}_0^1\text{n}$ and (ii) ${}_2^4\text{He} + {}_7^{14}\text{N} \Rightarrow {}_8^{17}\text{O} + {}_1^1\text{H}$

Given $M\left({}_1^3\text{H}\right) = 3.0169982$, $M\left({}_1^2\text{H}\right) = 2.0147361$, $M\left({}_2^4\text{He}\right) = 4.0038727$ $M\left({}_0^1\text{n}\right) = 1.0089832$, $M\left({}_7^{14}\text{N}\right) = 14.003074$, $M\left({}_8^{17}\text{O}\right) = 16.999133$ and $M\left({}_1^1\text{H}\right) = 1.007825$ all in a.m.u. Indicate also if the reaction is exoergic or endoergic.

Soln. We have $Q = (\text{mass of reactants} - \text{mass of products})\times c^2 = \Delta m.c^2$

(i) Here, $$\Delta m = \left[\left\{M\left({}^3\text{H}\right) + M\left({}^2\text{H}\right)\right\} - \left\{M\left({}^4\text{He}\right) + M\left({}^1\text{n}\right)\right\}\right]$$

$$= \left[(3.0169982 + 2.0147361) - (4.0038727 + 1.0089832)\right]\text{a.m.u.} = 0.0188784\text{ a.m.u.}$$

$$\Rightarrow Q = 0.0188784\times 931.48\text{ MeV} = +17.57\text{ MeV. Reaction is exoergic.}$$

(ii) Here, $$\Delta m = \left[\left\{M\left({}^4\text{He}\right) + M\left({}^{14}\text{N}\right)\right\} - \left\{M\left({}^{17}\text{O}\right) + M\left({}^1\text{H}\right)\right\}\right]$$

$$= \left[(4.003872 + 14.003074) - (16.999133 + 1.007825)\right]\text{a.m.u.}$$

$$= (18.0006946 - 18.006958)\text{ a.m.u.} = -0.000012\text{ a.m.u.}$$

$$= -0.000012\times 931.48\text{ MeV} = -0.11178\text{ MeV}$$

So, the reaction is endoergic.

12. Assume that 4 hydrogen nuclei are used to form a helium nucleus in the sun to provide the total energy to it. Calculate (i) the energy released when 1 gm-atom of hydrogen is fused to helium, (ii) how much hydrogen is to be converted to helium in the sun per second, given: mass of hydrogen nucleus = 1.00813 a.m.u, mass of He-nucleus = 4.00386 a.m.u, solar constant = 1.35 kW.m^{-2}. earth-sun distance = 1.5×10^8 km and there is no other loss in the energy radiated by the sun.

Soln. (i) Mass of H-nucleus = 1.00813 a.m.u; Mass of He-nucleus = 4.00386 a.m.u.

Therefore, Mass difference on fusion = $(4\times 1.00813 - 4.00386)$ a.m.u. = $(4.03252 - 4.00386)$ a.m.u. = 0.02866 a.m.u.

Therefore, Energy released = 0.02866×931 MeV = 26.68 MeV. This amount is released due to fusion of 4 H-atom into helium.

Therefore, Energy released due to fusion of 1 gm-atom of H

$= (26.68 \times 6.02 \times 10^{23}/4)$ MeV $= 40.15 \times 10^{23}$ MeV

(ii) Energy released on fusion of 4 H-atom, $E = 0.02866$ a.m.u.$\times c^2 = 0.02866 \times 1.66 \times 10^{-27}$ kg

$= 0.02866 \times 1.66 \times 10^{-27} \times (3\times10^{8})^2$ J $= 4.28\times10^{-12}$ J

Therefore, Energy released per H-atom $= (4.28\times10^{-12}/4)$J $= 1.07\times10^{-12}$ J

Total energy radiated per sec. from the sun $= 1.35\times10^{3}\times4\pi\times\left(1.5\times10^{11}\right)^2 = 3.82\times10^{26}$ J

Therefore, number of H-atom required $= \dfrac{3.82\times10^{26}}{1.07\times10^{-12}} = 3.57\times10^{38}$

Therefore, required mass of hydrogen $= 3.57\times10^{38}\times1.00813\times1.66\times10^{-27}$ kg $= 5.97\times10^{11}$ kg

13. Consider the nuclear reaction $N^{14}(\alpha, p)O^{17}$. Mass of neutral atoms are given to be

$_1H^1 = 1.007825$ a.m.u. $\quad _2He^4 = 4.002603$ a.m.u.

$N^{14} = 14.003074$ a.m.u., $\quad O^{17} = 16.994131$ a.m.u.

determine the Q.value of the reaction in MeV. Calculate the threshold K.E. of α-particle for the reaction.

Soln. Reaction $\quad {}_7N^{14} + {}_2He^4 \longrightarrow {}_1H^1 + {}_8O^{17}$

$\quad$ X $\quad$ x $\quad$ y $\quad$ Y

$$Q = \left(M_X + m_x - M_Y - m_y\right)c^2 = -(0.001279 \text{ a.m.u.})\, c^2 = -0.001279\times931.5 \text{ MeV}$$

$$E_x^{th} = -Q\left[\frac{m_y + M_Y}{m_y + M_Y - m_x}\right] = 1.535\,494 \text{ MeV}$$

14. Complete the following Nuclear Reactions

(i) ${}_{17}Cl^{35} + ? \rightarrow {}_{16}S^{32} + {}_2He^4$ (Ans. ${}_1H^1$) $\quad$ (ii) ${}_5B^{10} + ? \rightarrow {}_3Li^7 + {}_2He^4$ (Ans. ${}_0n^1$),

(iii) ${}_3Li^6 + ? \rightarrow {}_4Be^7 + {}_0n^1$ (Ans. ${}_1H^2$) $\quad$ (iv) ${}_{13}Al^{27} + {}_0n^1 \rightarrow {}_{12}Mg^{27} + ?$ (Ans. ${}_1H^1$),

(v) ${}_4Be^9 + {}_2He^4 \rightarrow ? + {}_0n^1$ (Ans. ${}_6C^{12}$)

(vi) ${}_3Li^7$ + proton → α − particle + ? (Ans. α − particle ${}_2He^4$)

(vii) Deuteron + Deuteron → proton + ? (Ans − tritium, ${}_1H^3$)

15. Complete the following disintegration reaction by substituting the proper nuclide or particle for the question mark in each case.

(i) ${}_{14}Si^{28}(n, p)?$ $\quad$ (ii) ${}_3Li^7(p, \gamma)?$ $\quad$ (iii) ${}_{13}Al^{27}(\alpha, p)?$ $\quad$ (iv) ${}_{11}Na^{23}(p, ?)\,{}_{10}Ne^{20}$

(v) ${}_{13}Al^{27}(?, \alpha)\,{}_{11}Na^{24}$ $\quad$ (vi) ${}_5B^{10}(?, \alpha)\,{}_3Li^7$ $\quad$ (vii) ${}_{11}Na^{23}(d, ?)\,{}_{13}Mg^{24}$

Soln. (i) ${}_{13}Al^{28}$ $\quad$ (ii) ${}_4Be^8$ $\quad$ (iii) ${}_{14}Si^{30}$ $\quad$ (iv) ${}_2He^4$ $\quad$ (v) ${}_0n^1$ (vi) ${}_0n^1$ $\quad$ (vii) ${}_0n^1$

16. Complete the following Reactions

(i) H^1 (n, γ) ? (ii) Li^7 (p, ?) Be^7 (iii) N^{14} (?, p) O^{17} (iv) ?(n, p) Na^{24}

Soln. (i) ${}_1H^2$, (ii) ${}_0n^1$, (iii) ${}_2He^4$ and (iv) ${}_{12}Mg^{24}$

17. An α-particle with K.E. $T_\alpha = 7.0$ MeV is scattered elastically by an initially stationary Li^6 nucleus. Find the K.E. of the recoil nuclers if the angle of divergence of the two particles is $\theta = 60°$

Soln. Initial momentum of α – particle is $\sqrt{2mT_\alpha}\,\hat{i}$

Final momenta are respectively $\vec{p}_\alpha$ and $\vec{p}_{Li}$.

Conservation of momentum reads

$\vec{p}_\alpha + \vec{p}_{Li} = \sqrt{2mT_\alpha}\,\hat{i} \quad \Rightarrow \quad p_\alpha^2 + p_{Li}^2 + 2p_\alpha p_{Li}\cos\theta = 2mT_\alpha$ ...(1)

Where θ is the angle between $\vec{p}_\alpha$ and $\vec{p}_{Li}$

Energy conservation: $\frac{p_\alpha^2}{2m} + \frac{p_{Li}^2}{2M} = T_\alpha \quad \Rightarrow \quad p_\alpha^2 + \frac{m}{M}p_{Li}^2 = 2mT_\alpha$ ...(2)

Where, m = mass of α, M = mass of Li^6

Subtract (2) from (1) we get $p_{Li}\left[\left(1-\frac{m}{M}\right)p_{Li} + 2p_\alpha\cos\theta\right] = 0 \quad p_{Li} \neq 0$

$\Rightarrow p_\alpha = -\frac{1}{2}\left(1-\frac{m}{M}\right)p_{Li}\sec\theta$

Now p_α, p_{Li} are both +ve number (bring magnitudes of vectors) we must have $-1 \le \cos\theta < 0$ if $m < M$

Thus we write, $\frac{p_{Li}^2}{2M}\left[1 + \frac{M}{4m}\left(1-\frac{m}{M}\right)^2\sec^2\theta\right] = T_\alpha$

Hence Recoil energy of Li nucleus is $\frac{p_{Li}^2}{2M} = \frac{T_\alpha}{1 + \frac{(M-m)^2}{4mM}\sec^2\theta}$

Putting, $\theta = 120°$, we get recoil energy of Li = 6 MeV.

18. Write missing symbols, denoted by x in the following nuclear reaction:

(a) $B^{10}(x, \alpha)Be^8 \rightarrow B^{10}(d, \alpha)Be^8$ (b) $O^{17}(d, n)x \rightarrow O^{17}(d, n)F^{18}$

(c) $Na^{23}(p, x)Ne^{20} \rightarrow Na^{23}(p, \alpha)Ne^{20}$ (d) $x(p, n)Ar^{37} \rightarrow Cl^{37}(p, n)Ar^{37}$

19. What amount of heat is liberated during the formation of one gram of He^4 from Deuterium H^2? What mass of the coal with calorific value of 30 kJ/g is thermally equivalent to the magnitude obtained?

Soln. Reaction is $H^2 + H^2 \rightarrow He^4 + Q$

$Q = 2\Delta_{H^2} - \Delta_{He^4}$ = (0.02820 – 0.00260)c^2= (0.02560 amu)c^2 = 23.8 MeV.

Hence the energy released in the 1gm of He^4 is $\frac{6.023\times10^{23}}{4}\times 23.8\times16.02\times10^{-13}$ Joule $= 5.75\times10^8$ kJ

This energy can be derived from $\frac{5.75\times10^8}{30000}$ kg $= 1.9\times10^4$ kg of coal

20. Making use of the tables of atomic masses, determine the energies of the following reaction.

(a) Li^7(p, n) Be^7 (b) Be^9 (n, r) Be^{10} (c) Li^7 (α, n)B^{10} (d) O^{16}(d, α)N^{14}

Soln. (a) $Li^7(p, n)Be^7$ Energy of reaction is $Q = (M_{Li^7} + M_{Be^7})C^2 + (M_P - M_n)C^2 = (\Delta_{Li^7} - \Delta_{Be^7})C^2 + \Delta_p - \Delta_n$

$= [0.01601 + 0.00783 - 0.01693 - 0.00867]$ amu $\times c^2 = -1.64$ MeV.

(b) $Be^9(n, \gamma)Be^{10}$: Mass of γ is taken as zero.

$$Q = (M_{Be^9} + M_n - M_{Be^{10}})c^2 = (\Delta_{Be^9} + \Delta_n - \Delta_{Be^{10}})c^2 = (0.01219 + 0.00867 - 0.01354) \text{ amu} \times c^2$$

$= 6.81$ MeV.

(c) $Li^7(\alpha, n)B^{10}$: $Q = (\Delta_{Li^7} + \Delta_\alpha - \Delta_n - \Delta_{B^{10}}) = (0.01601 + 0.00260 - 0.00867 - 0.01294)$ amu $\times c^2$

$= -2.79$ MeV

(d) $O^{16}(d, \alpha)N^{14}$: $Q = (\Delta_{O^{16}} + \Delta_d - \Delta_\alpha - \Delta_N^{14})c^2$

$= (-0.00509 + 0.01410 - 0.00260 - 0.00307)$ amu $\times c^2 = 3.11$ MeV

21. Find the velocity with which the products of the reaction $B^{10}(n, \alpha)Li^7$ come apart; the reaction proceeds via interaction of very slow neutrons with stationary Boron nuclei.

Soln. We have : $B^{10}(n, \alpha)Li^7$, $Q = (\Delta_{B^{10}} + \Delta_n - \Delta_\alpha - \Delta_{Li^7})c^2$

$= (0.01294 + 0.00867 - 0.00260 - 0.01601)$ amu $\times c^2 = 2.79$ MeV.

Since the incident neutron is very slow and B^{10} is stationary, the final total momentum must also be Zero. So the reaction products must emerge in opposite direction. If their speeds are respectively, v_α and V_{Li}

then $4V_\alpha = 7V_{Li}$ and $\frac{1}{2}(4V_\alpha^2 + 7V_{Li}^2) \times 1.672 \times 10^{-24} = 2.79 \times 1.602 \times 10^{-6}$.

So, $\frac{1}{2} \times 4V_\alpha^2 \left(1 + \frac{4}{7}\right) = 2.70 \times 10^{18} cm^2/s^2 \Rightarrow V_\alpha = 9.27 \times 10^6 m/sec$

$\Rightarrow V_{Li} = 5.3 \times 10^6$ m/sec.

22. Protons striking a stationary Lithium target activate a reaction $Li^7(p, n)Be^7$. At what value of the proton KE can the resulting neutron be stationary?

Soln. $Q = -1.64$ MeV.

Conservation of Momentum: $p_p = p_{Be}$ (since Initial Li and final neutron are both at rest)

$$\Rightarrow \frac{p_p^2}{2m_p} = \frac{p_{Be}^2}{2m_{Li}} + 1.64 \quad \Rightarrow \frac{p_p^2}{2m_p}\left(1 - \frac{m_p}{m_{Be}}\right) = 1.64$$

Hence, $T_p = \frac{p_p^2}{2m_p} = \frac{7}{6} \times 1.64 \text{ MeV} = 1.91 \text{ MeV}$

23. A beam of proton (of a certain energy) equivalent to a current of 1.6 mA is incident uniformly on a ^{56}F target containing 10^{22} atom per m². S0 that the following reaction take places, $p + {}^{56}Fe \longrightarrow n + {}^{53}Co$ If cross section for the reaction is 1 barn. Calculate the number of neutrons produced per sec.

Soln. $^{56}Fe + p \longrightarrow {}^{53}Co + n$ Given : $I = 1.6 \text{ mA} = 1.6 \times 10^{-3}$ A

$\Rightarrow q = it \quad \Rightarrow \frac{q}{t} = i \qquad$ Since, $q = ne \Rightarrow \frac{ne}{t} = i$

Number of incident particle per unit time is $N_0 = \frac{n}{t} = \frac{i}{e} = \frac{1.6 \times 10^{-3}}{1.6 \times 10^{-19}} = 10^{16}/sec$

No. of particle which interact with target is $N_0 - N = N_0 - N_0 e^{-\sigma nx} = N_0\{1-e^{-\sigma nx}\}$

where, $nx = 10^{22}$ atom per m².

$$\Rightarrow N_0 - N = 10^{16}\left\{1-e^{-\{10^{-6}\}}\right\} = 10^{10} = \text{Number of neutron produced per second.}$$

24. A 16 μA beam of a particle having cross-sectional area of $10^{-4}\ m^2$ is incident on a Rh target of thickness 1 μm. This produces neutrons through the reaction $\alpha + {}^{100}Rh \longrightarrow Pd + 3n$

(1) Number of α particle hitting the target per second is:

(a) $0.5\times10^{+14}$ (b) 1×10^{14} (c) 2×10^{20} (d) 4×10^{20}

Soln. $\frac{n}{t} = \frac{i}{2e} = \frac{16\times10^{-6}}{2\times1.6\times10^{-19}} = 0.5\times10^{14}$

25. Consider the decay process $\tau^- \to \pi^- + \nu_\tau$ in the rest frame of the τ^-. The masses of τ^-, π^- and ν_τ are μ_τ, μ_π and zero respectively. Find the energy and velocity of π.

Soln. Kinetic energy $= -Q\left[\frac{\text{Sum of masses of all particles involved in the reaction}}{2\times\text{mass of target}}\right]$

Where $Q = [m_i - m_f]c^2 = [m_\tau - m_\pi - 0]c^2$

Kinetic energy $= -(m_\tau - m_\pi)c^2\left[\frac{m_\tau + m_\pi}{2m_\tau}\right] = \left[\frac{-m_\tau^2 - m_\tau m_\pi + m_\pi m_\tau + m_\pi^2}{2m_\tau}\right]c^2 = \frac{(-m_\tau^2 + m_\pi^2)}{2m_\tau}c^2$

The energy of π^- is

E_{th} = Kinetic energy + rest mass energy

$$= \frac{[-m_\tau^2 + m_\pi^2]c^2}{2m_\tau} + m_\tau c^2 = \frac{(m_\tau^2 + m_\pi^2)c^2}{2m_\tau}$$

26. At a centre-of-mass energy of 5 MeV, the phase describing the elastic scattering of a neutron by a certain nucleus has the following values, $\delta_0 = 30°$, $\delta_1 = 10^0$. Assuming all other phase shifts to be negligible, plot $d\sigma/d\Omega$ as a function of scattering angle. Explicitly calculate $d\sigma/d\Omega$ at 30°, 45° and 90°. What is the total cross section σ?

Soln. The differential cross section is given $\frac{d\sigma}{d\Omega} = \frac{1}{k^2}\left|\sum_{\ell=0}^{\infty}(2\ell+1)e^{i\delta_\ell}\sin\delta_\ell P_\ell(\cos\theta)\right|^2$

Supposing only the first and second terms are important, we have

$$\frac{d\sigma}{d\Omega} \approx \frac{1}{k^2}\left|e^{i\delta_0}\sin\delta_0 + 3e^{i\delta_1}\sin\delta_1\cos\theta\right|^2$$

$$= \frac{1}{k^2}\left[\left(\cos\delta_0\sin\delta_0 + 3\cos\delta_1\sin\delta_1\cos\theta\right) + i\left(\sin^2\delta_0 + 3\sin^2\delta_1\cos\delta\right)\right]^2$$

$$= \frac{1}{k^2}\left[\sin^2\delta_0 + 9\sin^2\delta_1\cos^2\theta + 6\sin\delta_0\sin\delta_1\cos(\delta_1 - \delta_0)\cos\theta\right]^2$$

$$= \frac{1}{k^2}\left[0.25 + 0.27\cos^2\theta + 0.49\cos\theta\right]$$

where 'k' is the wave number of the incident neutron in the centre of mass frame. Assume that the mass of the nucleus is far larger than that of the neutron m_n. Then

$$k^2 \approx \frac{2m_n E}{\hbar^2} = \frac{2m_n c^2 E}{(\hbar c)^2} = \frac{2\times 938\times 5}{197^2\times 10^{-30}} = 2.4\times 10^{29}\,m^{-2} = 2.4\times 10^{25}\,m^{-2}$$

θ	0^0	30^0	45^0	90^0	180^0
$k^2\frac{d\sigma}{d\Omega}$	1	0.88	0.73	0.25	0
$\frac{d\sigma}{d\Omega}\times 10^{26}\,(cm^2)$	4.2	3.7	3.0	1.0	0

The total cross section is:

$$\sigma = \int \frac{d\sigma}{d\Omega} = \frac{2\pi}{k^2}\int_0^{\pi}\left(0.25 + 0.49\cos\theta + 0.27\cos^2\theta\right)\sin\theta\, d\theta$$

$$= \frac{4\pi}{k^2}\left(0.25 + \frac{1}{3}\times 0.27\right) = 1.78\times 10^{-25}\,cm^2 \approx 0.18\text{ barn}$$

27. Neutrons of 1000 eV kinetic energy are incident on a target composed of carbon. If the inelastic cross section is 400×10^{-24} cm², what upper and lower limits can you place on the elastic scattering cross section?

Soln. At 1 keV kinetic energy, only s-wave scattering is involved. The phase shift δ must have a positive imaginary part for inelastic process to take place. The elastic and inelastic cross sections are respectively given by

$$\sigma_e = \pi\lambda^2\left|e^{2i\delta} - 1\right|^2, \quad \sigma_{in} = \pi\lambda^2\left(1 - \left|e^{2i\delta}\right|^2\right)$$

The reduced mass of the system is $\mu = \frac{m_n m_e}{m_e + m_n} \approx \frac{12}{13} m_n$

For E = 1000 eV, $$\lambda = \frac{\hbar}{\sqrt{2\mu E}} = \frac{\hbar c}{\sqrt{2\mu c^2 E}} = \frac{197}{\sqrt{2\times\frac{12}{13}\times 940\times 10^{-3}}}\,m = 150\text{ fm}$$

$$\pi\lambda = 707\times 10^{-24}\,cm^2$$

As, $$1 - \left|e^{2i\delta}\right|^2 = \frac{\sigma_{in}}{\pi\lambda^2} = \frac{400\times 10^{-24}}{707\times 10^{-24}} = 0.566$$

We have, $$\left|e^{2i\delta}\right| = \sqrt{1 - 0.566} = 0.659 \quad \Rightarrow e^{2i\delta} = \pm 0.659$$

Hence, the elastic cross section $\sigma_e = \pi\lambda^2\left|e^{2i\delta} - 1\right|^2$

has maximum and minimum values $(\sigma_e)_{max} = 707\times 10^{-24}(-0.659 - 1)^2 = 1946\times 10^{-24}\,cm^2$

$$(\sigma_e)_{min} = 707\times 10^{-24}(-0.659 - 1)^2 = 82\times 10^{-24}\,cm^2$$

28. Disregarding nucleon spin, set a lower bound on the elastic center of mass proton-neutron forward differential cross-section.

Soln. The forward p–n differential cross section is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{0^\circ} = |f(0)|^2 \ge |\mathrm{Im}\, f(0)|^2 = \left(\frac{k}{4\pi}\sigma_t\right)^2$$

where the relation between $\mathrm{Im}\, f(0)$ and σ_t is given by the optical theorem. As $k = p/\hbar$ we have

$$\left.\frac{d\sigma}{d\Omega}\right|_{0^\circ} \ge \left(\frac{pc}{4\pi\hbar c}\sigma_t\right)^2 = \left(\frac{10^4 \times 40 \times 10^{-27}}{4\pi \times 1.97 \times 10^{-11}}\right)^2 \mathrm{m}^2 = 2.6 \times 10^{-24}\ \mathrm{cm}^2 = 2.6\ \mathrm{barn}$$

PRACTICE SET

1. Typical energies released in a nuclear fission and a nuclear fusion rection are respectively:
(a) 50 MeV and 1000 MeV
(b) 200 MeV and 1000 MeV
(c) 1000 MeV and 50 MeV
(d) 200 MeV and 10 MeV

2. An atomic bomb consisting of ^{235}U explodes and release an energy of 10^{14} J. It is known that each ^{235}U which undergoes fusion release 3 neutrons and about 200 MeV of energy. Further, only 20% of the ^{235}U atoms in the bomb undergo fission. The total number of neutrons released is about:
(a) 4.7×10^{24}
(b) 9.7×10^{24}
(c) 1.9×10^{25}
(d) 3.7×10^{25}

3. The condition for spontaneous fission of a nucleus is :
(a) $\frac{Z^2}{A} < 50$
(b) $\frac{Z^2}{A} > 45$
(c) $\frac{Z}{A} > 45$
(d) $\frac{Z}{A} < 45$

4. Muons are produced through the annihilation of particle a and its antiparticle, namely the process $a + \bar{a} \to \mu^+ + \mu^-$. A muon has a rest mass of 105 MeV/c^2 and its proper life time is $2\,\mu s$. If the center of mass energy of the collision is 2.1 GeV in the laboratory frame that coincides with the center-of-mass frame, then the fraction of muons that will decay before they reach a detector placed 6km away from the interaction point is
(a) e^{-1}
(b) $1 - e^{-1}$
(c) $1 - e^{-2}$
(d) e^{-10}

5. A spin-1/2 particle A undergoes the decay $A \to B + C + D$ where it is known that B and C are also spin-1/2 particles. The complete set of allowed values of the spin of the particle D is
(a) $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$
(b) 0, 1
(c) $\frac{1}{2}$ only
(d) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots$

ANSWER KEY					
Questions	1	2	3	4	5
Option	(d)	(b)	(b)	(d)	(d)

Elementary Particles

Introduction :

It is a dream for physicists to be able to explain the material world, the physical universe entirely. This has driven them continuously to explore the fundamental entities of matter was atom then nucleus, neutron and protons and now the quarks. In this present chapter we will read about all elementary particle their spin charge, isospin, mass etc.

Classification of elementary particles:

The elementary particles may be classified in a number of different ways depending on their masses, interaction, statistics etc. Commonly, however, they are classified into such categories as

- Elementary particles
  - Photon
  - Laptons
    - (i) electrons
    - (ii) electrons-neutrino
    - (iii) muon
    - (iv) muon neutrino
    - (v) tau
    - (vi) tau-neutrino
  - Hadrons
    - Mesons
      - (i) Pions ($\pi \pm 0$)
      - (ii) Kaons ($k \pm 0$)
      - (iii) ηmeson (°)η
    - Baryons
      - Nucleons
        - (i) proton
        - (ii) neutrons
      - Hyperons
        - (I) Lambda (Λ^0)
        - (ii) sigma ($\Sigma^{\pm 0}$)
        - (iii) xi (Ξ^{-0})
        - (iv) omega (Ω^-)

"Some elementary particle and their properties:

Category	Particles Name	Symbols	Charge	Spin	B	L	s	I_3	Y	Quark structure
Laptons	Electrons	e^-	−1	½	0	1	0		0	
	Electron neutrino	ν_e	0	½	0	1	0		0	
	Mouns	$\mu-$	−1	½	0	1	0		0	
	Moun Neutrino	ν_μ	0	½	0	1	0		0	
	Tauons	τ	−1	½	0	1	0		0	
	Tauons Neutrino	ν_τ	0	½	0	1	0		0	
Mesons	Pions	π^+	+1	0	0	0	0	+1	0	$u\bar{d}$
		π^0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$
		π^-	−1	0	0	0	0	−1	0	$d\bar{u}$
	Kaons	k^+	+1	0	0	0	+1	-½	+1	$u\bar{s}$
		k^0	0	0	0	0	+1	-½	+1	$d\bar{s}$
		k^-	−1	0	0	0	−1	½	−1	$s\bar{u}$
	Eta	η^0	0	0	0	0	0	0	0	$\frac{1}{\sqrt{6}}(u\bar{u}+d\bar{d}-2s\bar{s})$
Nucleons	Proton	P	+1	½	1	0	0	½	1	uud
	Neutron	n	0	½	1	0	0	-½	1	udd
Hyperons	Lambda	Λ^0	0	½	1	0	−1	½	0	uds
	Sigma	Σ^+	+1	½	1	0	−1	+1	0	uus
		Σ^0	0	½	1	0	−1	0	0	uds
		Σ^-	−1	½	1	0	−1	0	0	dds
	Xi	Ξ^0	0	½	1	0	-2	½	−1	uss
		Ξ^-	−1	½	1	0	-2	-½	-1	dss
	Omega	Ω^-	-1	½	1	0	−3	0	−2	sss

- Conservation laws in elementary particle reactions:
- **Exact conservation laws:**
  (i) Conservation of linear momentum
  (ii) Conservation of angular momentum
  (iii) Conservation of charge
  (iv) Conservation of baryon number
  (v) Conservation of lepton number

- **Approximate conservation laws:**
  (i) Isospin or isotopic spin(I):
  It arose however from the idea that pairs of particles like nucleons and triplets like pions hardly differ in their mass and may be considered as isotopes and that their charges, differing from each other by unity, suggest space qunatization similar to electron spin and orbit in a magnetic field.

- **A multiplet number** (M) is defined as the number of their different charge states. For instance, for nucleons i.e. protons or neutrons - the multiplet number M = 2. Similarly, for the triplet of pions, M

- Isospin is treated as a vector $\bar{I}$ of magnitude $\sqrt{I(I+1)}$, like angular momentum, but I is dimensionless. Its component along Z-axis, is given by I_3 which have the allowed values.

  $I, (I-1), (I-2), \ldots\ldots\ldots\ldots -I$

- For nucleons $(M=2), I=(M-1)/2=\frac{1}{2}$ and the values of I_3 are $+\frac{1}{2}$ and $-\frac{1}{2}$; $I_3=+\frac{1}{2}$ is assigned to proton, $I_3=-\frac{1}{2}$ to neutron.

- For pions $(M=3), I=(M-1)/2=1$. Hence, $I_3=+1, 0, -1$. $I_3=+1$ is assigned to π^+, $I_3=0$ to π^0 and $I_3=-1$ to π^-

- Isospin is conserved in strong interactions but is violated in electromagnetic and weak interaction. The Z-component of isospin, I_3- is conserved in strong and electromagnetic interaction and not in weak interaction.

(ii) Hypercharge(Y): It is defined as double of the average charge $\bar{Q}$ of the multiplet.

$$Y=2\bar{Q} \quad \Rightarrow \bar{Q}=Y/2 \quad \Rightarrow \boxed{Q=I_3+Y/2}$$

- For any strong and electromagnetic interaction, the hypercharge is conserved, i.e. remains invariant. But it need not be conserved in weak interaction.

Example: $p+p \rightarrow \Lambda^0+K^0+p+\pi^+$

Hypercharge: $1+1=0+1+1+0 \quad \Rightarrow \Delta Y=0$

(iii) Strangeness number (S): It is defined as the difference of the hyper change Y and the baryon number B.

$$S=Y-B \quad \Rightarrow Y=S+B$$

i.e., the hyperchange is the sum of the baryon number and the strangeness number

Therefore, $$Q=I_3+\frac{B+S}{2}$$

- Strangeness number S is conserved in strong and electromagnetic reactions. For a weak interactions, $\Delta S=0$ or ± 1

(iv) Parity:

When particle like neutrions are emitted during radioactive decay, they show a preferred spin direction. If a neutrino spins in the direction at which a right-handed screw advances, it is said to posses a helicity +1; if however the spin is in the direction of a left-handed screw, the helicity is -1. As the parity P is related to the spin J, the two quantum numbers are usually combined and is symbolised by J^P. So, $\left(\frac{1}{2}\right)^+$ means the J-value is $\frac{1}{2}$ and P=+1; $\left(\frac{1}{2}\right)^-$ indicates $J=\frac{1}{2}, P=-1$.

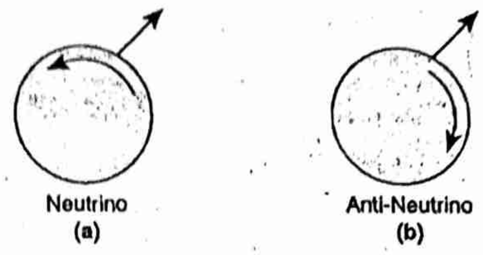


Figure: Helicity of neutrino and antineutrino

- In strong and electromagnetic interactions, parity is conserved but it is not conserved in weak interaction.

(v) Charge conjugation:

Charge conjugation means reversal of the sings of all types of charge i.e. electric, baryonic and laptonic of the particles (Figure below). If a physical law holding for particles also holds for corresponding antiparticles, the principle of charge conjugation is said to be valid.

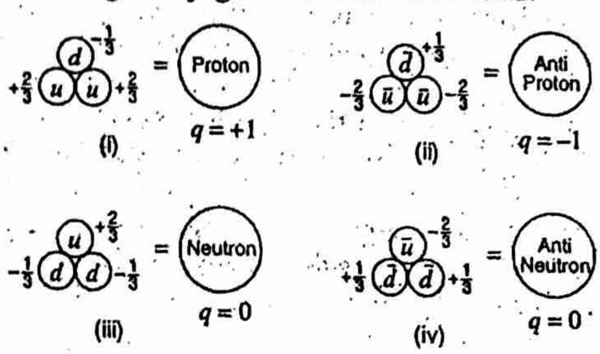


Figure: Formation of proton, antiproton and neutron, antineutron by charge conjugation

- Strong and electromagnetic interactions are charge conjugate invariant. But the weak intervation like $\beta-$decay does not obey charge conjugation.

(vi) Time reversal:

The operation T i.e., time-reversal means replacing the time 't' by –t in all equations of motion i.e., reflection of time axis at the origin of time coordinate in relativistic space-time continum. It is thus, like the parity operation, a discrete change.

- T-operation consists in reversing the signs of momenta $\left(\bar{p} = d\bar{r}/dt\right)$ and angular momenta $\left(\bar{L} = \bar{r} \times \bar{p}\right)$.

  T also transforms the wave function to its complex conjugate. If 'T' is conserved i.e., time-reversal invariance occurs, then the reversed equation of motion is also a valid equation of motion of the system concerned. All the known fundamental equations of motion are invariant in time-reversal.

- Strong and electromagnetic interactions are invariant under time-reversed transformation.

- **CPT Theorem:**

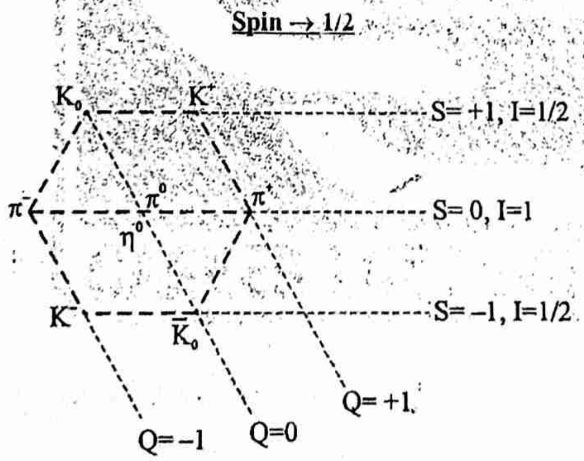
This is an exact conservation law. It states that all interactions in nature are invariant under joint operations of charge conjugation (C), inversion of space coordinates at origin, i.e. parity (P) and reversal of time (T). The order of operations is immaterial.

The invariance of CPT transformation implies that if any interaction is not invariant under any one of C, P and T operations, its effect gets compensated by the joint effect of the other two.

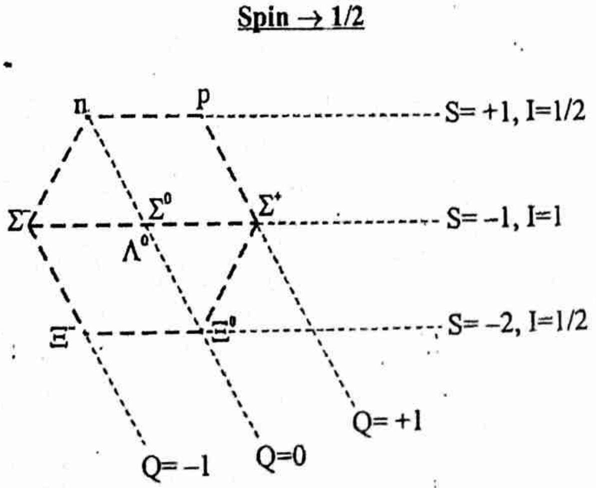
- **Conserved quantities in different type of reactions:**

Conserved Quantities	Strong	Electromagnetic	Weak
1. Charge	✓	✓	✓
2. Linear Mom.	✓	✓	✓
3. Relativistic En	✓	✓	✓
4. Spin	✓	✓	✓
5. Le	✓	✓	✓
6. Lu	✓	✓	✓
7. Li	✓	✓	✓
8. Baryon No. B	✓	✓	✓
9. I_z	✓	✓	×
10. I	✓	×	×
11. S	✓	✓	×
12. Y = B + S	✓	✓	×
13. Parity (P)	✓	✓	×
14. Charge conjugation	✓	✓	×
15. Time reversion (T)	✓	✓	✓
16. CP	✓	✓	×
17. CPT	✓	✓	✓

- **Meson Octate:**



- **Baryon Octate:**



- **Relationship between particles and antiparticles:**

(a) Mass	→	Same
(b) Spin	→	Same
(c) Charge	→	Same but opposite in sign.
(d) Mag. moment	→	Same in magnitude but opposite in sign.
(e) Mean life time of free decay	→	Same
(f) Annihilation	→	In pairs
(g) Creation	→	In pairs
(h) Total isotopic spin	→	same
(i) Third component of isospin(I_3)	→	Same in mag. but opposite in sign.
(j) Intrinsic parity	→	Same for bosons and opposite for Fermions.
(k) Strangeness q.No. (.S)	→	Same in mag. but opposite in sign.
(l) Lepton no. (L)	→	Same in mag. but opposite in sign.
(m) Baryon no. (B)	→	Same in mag. but opposite in sign.
(n) Hypercharge (Y)	→	Same in mag. but opposite in sign.

The particle and antiparticle have same symbol except a bar over particle for antiparticle.

- **Concept of antiparticle by Dirac:**

$$E = \pm\sqrt{p^2c^2 + m_0^2c^4}$$

So, positive solution for particle are negative energy state are filled like a sea.

The negative solution of Schrodinger equation is also possible. But Dirac said that all the negative energy states are already filled. If electron given enough energy it can come out the filled negative state to positive energy.

The vaccent state in negative energy behaves like holes → antiparticles

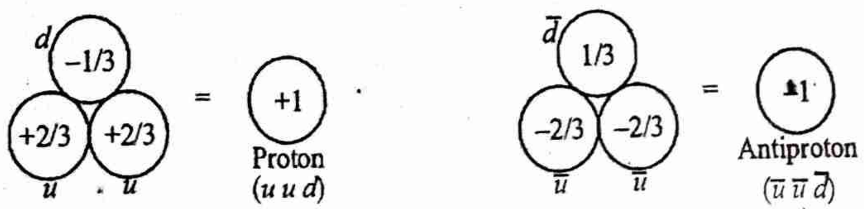
- **Quark Model:** The elementary particles can be conceived (as far as isospin and hyper charge are concerned) as being built out of combination of quarks. The original three quarks were called up (Symbol u), down (d) and strange (s)

Each quarks has an anti-quark associated with it $(\bar{u}, \bar{d}, \bar{s})$. Since a quark and its anti-quark have opposite quantum numbers. They can be created from energy. In the reverse process, a quark and its anti-quark annihilate and give energy.

Thus, energy $\rightarrow u + u$

$d + d \rightarrow$ energy

Quark models for proton and antiproton.



Coloured Quark: Quarks and antiquarks have an additional property of some kind that can be manifested in a total of six different ways, rather as electric charge is a property that can be manifested in the two different ways that have come to be called positive and negative. In the case of quarks, this property became known as colour and its three possibilities were called red, green and blue. The antiquark colours are antired, antigreen and antiblue.

According to the colour hypothesis, all three quarks in a baryon have different colours which satisfies the exclusion principle since all are then a different states even if two or three are otherwise identical

The rules for combining colours are the following:

- A colour and its anticolour cancel out. This is called colourless or white.
- All three colours or all three anticolours in combination cancel out and give colourless
- All hadrons are colourless.

Mesons consists of a quark-antiquark pair of a particular colour and its anticolour. Baryons are made up of three quarks, one of each colour. Thus mesons and baryons are white or colourless.

Charm Bottom and TOP Quarks: Besides the three quarks (*u*, *d*, *s*) three more quarks are suggested in order to have a significant analysis of the symmetries. These are charm (*c*), top (*t*) and bottom (*b*). Thus over all there are six quarks. The various characteristics of these are given in the following table:

Quark	I	I_3	B	S	Y	Q	*spin*	C	E	B	T	*Mass (GeV)*
Up(*u*)	1/2	+1/2	1/3	0	1/3	2/3	½	0	0	0	0	0.39
Down(*d*)	1/2	−1/2	1/3	0	1/3	−1/3	½	0	0	0	0	0.39
Strange(*s*)	0	0	1/3	−1	−2/3	−1/3	½	0	0	0	0	0.51
Charm(*c*)	0	0	1/3	0	1/3	2/3	½	1	1	0	0	1.55
Top(*t*)	0	0	1/3	0	1/3	−1/3	½	0	0	0	1	5.4
Bottom(*b*)	0	0	1/3	0	1/3	2/3	½	0	0	1	0	20

- **Few examples of particle quark structure:**

$K_0 = \bar{s}d;\ K^+ = \bar{s}u;\ \pi^- = d\bar{u}, \pi^0$ & η^0 may be $u\bar{u}$ and $d\bar{d}$

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right) \qquad \text{parity} = -1$$

$$\eta^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \qquad \text{parity} = +1$$

$$\pi^+ = u\bar{d};\ K^- = s\bar{u};\ \bar{K}_0 = s\bar{d}$$

$$n = udd;\ p = uud$$

$$\Sigma^- = sdd;\ \Sigma^+ = suu;\ \Xi^- = ssd;\ \Xi^0 = ssu;\ \Omega^- = sss$$

λ^0 and Σ^0 may both be sud

$$\Sigma^0 = \frac{1}{\sqrt{2}}(sud + sud) \longrightarrow \text{parity} = +1$$

$$\lambda^0 = \frac{1}{\sqrt{2}}(sud - sud) \longrightarrow \text{parity} = -1$$

Baryon Resonances :

Particle	I	I_3	B	S	Y	Structure
Δ^{++}	$3/2$	$3/2$	1	0	1	uuu
Δ^{+}	$3/2$	$1/2$	1	0	1	uud
Δ^{0}	$3/2$	$-1/2$	1	0	1	udd
Δ^{-}	$3/2$	$-3/2$	1	-1	0	uus
Σ^{*+}	1	1	1	-1	0	uds
Σ^{*0}	1	0	1	-1	0	dds
Σ^{*-1}	1	-1	1	-1	0	dds
Ξ^{*0}	$1/2$	$1/2$	1	-2	-1	uss
Ξ^{*-}	$1/2$	$-1/2$	1	-2	-1	dss
Ω^{-}	0	0	1	-3	-2	sss

The four fundamental force:

In nature, there are four different types of fundamental interactions. They are

1. The gravitational force :

The gravitational force is the oldest known force. It holds us to the surface of the earth and holds the entire unverse together. It is a long-range force varying as $\frac{1}{r^2}$. It has life time $\tau = 10^{16}$ sec

2. The electromagnetic force:

The electromagnetic force was the second force known. In fact it was originally two forces the electric force and magnetic force. The electromagnetic force hold the atoms, molecules, solid and liquid together. It is also long rang force varying as $\frac{1}{r^2}$. It has a life time, $\tau = 10^{-20}$ sec

3. Weak interaction force:

This fundamental interaction involves leptons and hedrons. The β-decay of radioactive nuclei and decays of strong particles are typicaly of weak interaction. It has a life time, $\tau = 10^{-10}$ sec

4. Strong interaction force :

The strong nuclear force is responsible for holding the nucleons together. It is the strongest of all the forces but is a very short range force. Its effect occur within a distance of about 10^{-15} m. The strong nuclear acts only on the hadrons. It has a life time, $\tau = 10^{-23}$ sec.

Kinematics of high energy collisions (Relativistics):

Many elementary particles are the product of high energy collision. We shall now study the kinematics of such collision, restricting to those that result in the production of two product particles only. But it can be easily generalised. For compactness, we shall use a system of unit where the light velocity a would be taken as unity, i.e. $c = 1$, so that mc^2 is denoted is denoted by the mass m, the energy pc momentum p etc.

Let us take reaction

$$A_1 + A_2 \rightarrow A_3 + A_4 \qquad \text{... (1)}$$

The concervation principles of energy and momentum

$$W_1 + W_2 = W_3 + W_4 \qquad \text{... (2)}$$

And $$P_1 + P_2 = P_3 + P_4 \qquad \text{... (3)}$$

where W's represent total energies and P's the momenta of the individual particles.

Using relatívistics relations for different particles, we get

$$W_n^2 = P_n^2 + M_n^2, n = 1,2,3,..... \qquad \text{... (4)}$$

From relativisty again the total energy W and the components of momentum vector $\vec{P}$ constitute a four vector $P_\mu = (W, P)$ and these scalar product $P_\mu P_\mu$ in 4-space is an invariant at a particular time.

$$\therefore \quad P_\mu P_\mu = W_\cdot^2 - P^2 = \text{invariant} \qquad \text{... (5)}$$

P^2 being the square of 3d-momentum vector.

Therefore, for a system of particles, in initial on final state.

$$P_\mu P_\mu = (\Sigma W_n)^2 - (\Sigma P_n)^2 = \text{invariant} \qquad \text{... (6)}$$

In reference to the reaction (1), therefore, we have

$$(P_\mu P_\mu)_{initial} = (W_1 + W_2)^2 - (P_1 + P_2)^2$$

$$= \left(\sqrt{P_1^2 + m_1^2} + \sqrt{P_2^2 + m_2^2}\right)^2 - (P_1 + P_2)^2 \qquad \text{... (7)}$$

In Lab-frame, assuming the target A_2 to be at rest $P_2 = 0$. The kinetic energy T_1 of the incident particle A_1 is then

$$T_1 = W_1 - m_1 = \sqrt{P_1^2 + m_1^2} - m_1$$

Therefore, from (8), we obtain the relation,

$$(P_\mu P_\mu)_{\text{initial}} = \left(\sqrt{P_1^2 + m_1^2} + m_2\right)^2 - P_1^2$$

$$= m_1^2 + m_2^2 + 2m_2\sqrt{P_1^2 + m_1^2}$$

$$= m_1^2 + m_2^2 + 2m_2(m_1 + T_1)$$

$$= (m_1 + m_2)^2 + 2m_1 T_1$$

Threshold energy:

At the threshold, A_3 and A_4 are produced with zero momentum, i.e. we then have $P_3 = P_4 = 0$ using (8)

$$(P_\mu P_\mu)_{final} = (m_3 + m_4)^2 \qquad \text{... (9)}$$

Substituting $m_i = m_1 + m_2$ and $m_f = m_3 + m_4$ and exploting the invariance property we get from (8) and (9)

$$m_i^2 + 2m_2 T_1 = m_f^2$$

$$\Rightarrow \quad \boxed{T_1 = (m_f^2 - m_i^2)/2m_2} \qquad \text{... (10)}$$

Again, Q = value of the reaction is given by

$$Q = m_1 + m_2 - m_3 - m_4 = m_i - m_f \qquad \text{... (11)}$$

Threshold energy,

$$E_{th} = T_1 = \frac{(m_f + m_i)(m_f - m_i)}{2m_2} \ (\text{using } (10))$$

$$= -\frac{Q}{2m_2}(m_i + m_f) = \frac{Q}{2m_2}(Q - 2m_i)$$

$$\boxed{E_{th} = Q\left[\frac{Q}{2m_2} - \frac{m_1 + m_2}{m_2}\right]} \quad \text{...(12)}$$

1. **Non-Relativistic collision:** Hence, $Q << m_1 \,(or\ m_2)$ so, from (12)
We obtain

$$\boxed{E_{th} = -Q\left(\frac{m_1 + m_2}{m_2}\right)} \quad \text{...(13)}$$

2. **Extreme relativistic case :** Here $Q >> m_1$ and also m_2 so that

$$\boxed{E_{th} = \frac{Q^2}{2m_2}} \quad \text{...(14)}$$

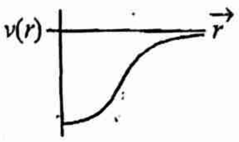
SOLVED PROBLEMS

1. The interaction potential between two quarks, separated by a distance r inside a nucleon be described by (a, b and b are positive constants) **[GATE 2006]**

(a) ae^{-br} (b) $\frac{a}{r} + br$ (c) $-\frac{a}{r} + br$ (d) $\frac{a}{r}$

Soln. Insider the nucleus potential vary like Saxon-woods mode. Only option (3) give this type of variation.

$$v(r) = -\frac{a}{r} + br$$



$$r \to 0, r(n) \to -\infty$$

$$r \to \infty, r(n) \to 0$$

Correct option is (c)

2. The strange baryon Σ^+ has the quark structure: **[GATE 2007]**
(a) uds (b) uud (c) uus (d) uss

Soln. Σ^+

$$\left.\begin{aligned} Q &= +1 = +\frac{2}{3} + \frac{2}{3} - \frac{1}{3} \\ B &= 1 = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} \\ S &= -1 = 0 + 0 - 1 \end{aligned}\right\} (uus)$$

Correct option is (c)

3. The isospin and the strangeness of Ω^- baryon are [GATE 2011]

(a) 1, −3 (b) 0, −3 (c) 1, 3 (d) 0, 3

Soln. Ω^-

Strangeness, $S=-3$

Isospin $I=\frac{M-1}{2}=\frac{1-1}{2}=0$

Correct option is (b)

4. The isospin (I) and baryon number (B) of the upquark is [GATE 2013]

(a) I = 1, B = 1 (b) I = 1, B = 1/3 (c) I = 1/2, B = 1 (d) I = 1/2, B = 1/3

Soln. Baryon number of upquark is $\frac{1}{3}$

Isospin number of upquark is $\frac{1}{2}$

Correct option is (d)

5. A K meson (with a rest mass of 494 MeV) at rest decays into a muon (with a rest mass of 106 MeV) and a neutrino. The energy of the neutrino, which can be taken to be massless, is approximately

(a) 120 MeV (b) 236 MeV (c) 300 MeV (d) 388 MeV [JEST 2013]

Soln. According to energy conservation.

The energy of the massless particle neutrino

$$E_\nu=\left(\frac{M_k^2-M_\mu^2}{2M_k}\right)MeV=\frac{(494)^2-(106)^2}{2\times 494}\approx 236$$

Correct option is (b)

6. Consider the following reaction involving elementary particles: [TIFR 2015]

(A) $\pi^-+p\to K^-+\Sigma^+$

(B) $K^-+p\to K^-+\rho^+$

Which of the following statements is true for strong interactions?

(a) (A) and (B) are both forbidden
(b) (B) is allowed but (A) is forbidden
(c) (A) is allowed but (B) is forbidden
(d) (A) and (B) are both allowed

Soln. (A) $\pi^-+p\to K^-+\Sigma^+$

$Q:-1\ \ +1\to -1\ \ +1$ $\quad \Delta Q=0$

$spin:0+\frac{1}{2}\to 0\ \ +\frac{1}{2}$ $\quad \Delta s=0$

$L:0\ \ 0\ \to 0\ \ 0$ $\quad \Delta L=0$

$B:0\ \ 1\ \to\ 0\ \ +1$ $\quad \Delta B=0$

$I_3:-1\ \ +\frac{1}{2}\to \frac{1}{2}\ \ \ +1$ $\quad \Delta I\neq 0$

(B) $K^- + p \to K^- + \rho^+$

$Q: -1 \quad +1 \to -1 \quad +1 \qquad \Delta Q = 0$

$spin: 0 + \frac{1}{2} \to 0 \quad +0 \qquad \Delta s \neq 0$

Therefore, (A) and (B) both are forbidden.

7. The decay $\mu^+ \to e^+ + \gamma$ is forbidden, because it violates **[GATE 2015]**
(a) momentum and lepton number conservations (b) baryon and lepton number conservations
(c) angular momentum conservation (d) lepton number conservation

Soln. $\mu^+ \to e^+ + \gamma$

$L: -1 \to -1 \qquad \Delta L = 0$

$B: 0 \to 0 \qquad \Delta B = 0$

Since momentum, lepton, banyon number is conserved.
Correct option is (c)

8. The reaction $e^+ + e^- \to \gamma$ is forbidden because, **[JEST 2015]**
(a) lepton number is not conserved (b) linear momentum is not conserved
(c) angular momentum is not conserved (d) charge is not conserved

Soln. $e^+ + e^- \to \gamma$

$L: -1 \quad +1 \to 0 \qquad \Delta L = 0$

$Q: +1 \quad -1 \to 0 \qquad \Delta Q = 0$

Since positron and electron rotation are equal magnitude and opposite direction. Therefore, angular momentum remain conserved in left hand side.
Therefore, angular momentum is not conserved. Therefore, correct option is (c).

9. Consider the decay of a pion into a muon and an anti-neutrino $\pi^- \to \mu^- + \bar{\nu}_\mu$ in the pion rest frame. $m_\pi = 139.6\,MeV/c^2$, $m_\mu = 105.7\,MeV/c^2$, $m_\nu \approx 0$. The energy (in MeV) of the emitted neutrino, to the nearest integer is _______ **[GATE 2013]**

Soln. According to energy conservation,

$$E_{\bar{\nu}} + E_\mu = M_\pi C^2$$

According to momentum conservation,

$$0 = P_{\bar{\nu}} + P_\mu \Rightarrow P_\mu = -P_{\bar{\nu}}.$$

$$E_{\bar{\nu}} = \sqrt{P_{\bar{\nu}}^2 C^2 + M_{\bar{\nu}}^2 C^4} = P_{\bar{\nu}} C \quad (\because M_\nu = 0)$$

$$\Rightarrow \quad M_\pi C^2 = \sqrt{P_\mu^2 C^2 + M_\mu^2 C^4} + P_{\bar{\nu}} C$$

$$\Rightarrow \quad \left((M_\pi C^2) - P_{\bar{\nu}} C\right)^2 = P_\mu^2 C^2 + M_\mu^2 C^4$$

$$\Rightarrow \quad E_\nu = \frac{\left(M_\pi^2 - M_\mu^2\right) C^2}{2 M_\pi} = \left[\frac{(139.6)^2 - (105.7)^2}{2 \times 139.6}\right] C^2 = 29.7\ MeV \simeq 30\ MeV$$

10. The nucleus of the atom $^9Bc_4$ consists of [GATE 2003]
(a) 13 up quarks and 13 down quarks (b) 13 up quarks and 14 down quarks
(c) 14 up quarks and 13 down quarks (d) 14 up quarks and 14 down quarks

Soln. $^9Bc_4$

Protons, $P \to 4 \;\; 4(u+u+d)$

Neutrons $n \to 5 \;\; (u+d+d)$

Therefore, number of up quarks = 13
Number of quarks = 14
Therefore, correct option is (b)

11. Choose the particle with zero Baryon number from the list given below. [GATE 2004]
(a) pion (b) neutron (c) proton (d) Δ^+

Soln. All Baryons have Baryon number + 1 and all others have 0 Baryon number.
Since pion in the mesons group.
Therefore, Baryon number of Pion is zero.
Correct option is (a)

12. The basic process underlying the neutron β-decay is [GATE 2010]
(a) $d \to u + e^- + \bar{V}_e$ (b) $d \to u + e^-$ (c) $s \to u + e^- + \bar{V}_e$ (d) $u \to d + e^- + \bar{V}_e$

Soln. In β-decay process one neutron decay into one proton and one electron and for parity conservation another massless particle electron neutrino will produce.

$$d \to u + e^- + \bar{v}_e$$

Correct option is (a)

13. The charm quark is assigned a charm quantum number $C = 1$. How should the Gellmann-Nishijima formula for electric charge be modified for four flavours of quarks? [NET June 2015]

(a) $I_3 + \frac{1}{2}(B - S - C)$ (b) $I_3 + \frac{1}{2}(B - S + C)$

(c) $I_3 + \frac{1}{2}(B + S - C)$ (d) $I_3 + \frac{1}{2}(B + S + C)$

Soln. In the general Gellmann formula is

$$Q = I_3 + \frac{B+S+C+T+B}{2}$$

Therefore, the quark C have C = 1, B = 0, T = 0

$$\boxed{Q = I_3 + \frac{B+S+C}{2}}$$

Correct option is (d)

14. A relativistic particle travels a length of 3×10^{-3} m in air before decaying. The decay process of the particle is dominated by: [GATE 2007]

(a) strong interactions (b) electromagnetic interactions.
(c) weak interactions (d) gravitational interactions

Soln. The traveling distance = $3 \times 10^{-3} m$
Therefore, the time taken by the particle to travel the distance before decay

$$t=\frac{3\times10^{-3}}{c}=\frac{3\times10^{-3}}{3\times10^{8}}=10^{-11}$$

Therefore, life time of particle is $T=10^{-11}$. Therefore, interaction is weak interaction.

Correct option is (c)

15. The decay process $n\rightarrow p^{+}+e^{-}+\overline{\nu}_e$ violates **[GATE 2013]**

(a) baryon number (b) lepton number (c) isospin (d) strangeness

Soln.

$n \longrightarrow$	p^{+} +	e^{-1} +	$\overline{\nu}_e$	
$B:1$	1	0	0	$\Delta B=0$
$L:0$	0	+1	−1	$\Delta L=0$
$I_3:-½$	½	0	0	$\Delta I=0\neq0$
$S:0$	0	0	0	$\Delta S=0$

Therefore, correct optinon is (c)

16. Which one of the following high energy processes is allowed by conservation laws? **[GATE 2014]**

(a) $p+\overline{p}\rightarrow\Lambda^0+\Lambda^0$ (b) $\pi^-+p\rightarrow\pi^0+n$

(c) $n\rightarrow p+e^-+\nu_e$ (d) $\mu^+\rightarrow e^-+\gamma$

Soln.

p +	$\overline{p} \longrightarrow$	Λ^0 +	Λ^0	
$Q:+1$	−1	0	0	$\Delta Q=0$
$spin:\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\Delta s=0$
$B:1$	−1	1	1	$\Delta B=-2\neq0$
$L:0$	0	0	0	$\Delta L=0$

Therefore, it is not allowed reaction because Baryon number is not conserved.

π^- +	$p \longrightarrow$	π^0 +	n	
$Q:-1$	+1	0	0	$\Delta Q=0$
$spin:0$	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$\Delta s=0$
$B:0$	1	0	1	$\Delta B=0$
$L:0$	0	0	0	$\Delta L=0$
$I_3:-1$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$\Delta I_3=0$

Therefore, it is a allowed reaction.

$n \longrightarrow$	p +	e^- +	ν_e	
$Q:0$	+1	−1	0	$\Delta Q=0$
$spin:½$	½	½	½	$\Delta s\neq0$

Therefore, it is not allowed reaction because spin is not conserved.

μ^+	$\longrightarrow$	e^-	$+\ \gamma$	
$Q:+1$		-1	0	$\Delta Q \neq 0$

It is not allowed reaction because charged is not conserved.

Correct option is (b)

17. Which one of the following three-quark states (qqqq), denoted by X, CANNOT be a possible baryon? The corresponding electric charge is indicated in the superscript **[GATE 2014]**

(a) X^{++} (b) X^{+} (c) X^{-} (d) X^{--}

Soln. Baryon have charges +2, +1, −1 but does not have −2 charge state.

Therefore, X^{--} does not represent a Baryon.

Correct option is (d)

18. Which one of the following nuclear processes is forbidden? **[GATE 2006]**

(a) $\bar{\nu}+p\rightarrow n+e^{+}$ (b) $\pi^{-}\rightarrow e^{-}+\nu_{e}+\pi^{0}$

(c) $\pi^{-}+p^{-}\rightarrow n+K^{+}+K^{-}$ (d) $\mu^{-}\rightarrow e^{-}+\bar{\nu}_{e}+\nu_{\mu}$

Soln.

$\bar{\nu}$	$+p$	$\longrightarrow n$	$+e^{+}$	
$Q:0$	$+1$	0	$+1$	$\Delta Q=0$
$spin:-½$	$+½$	$½$	$-½$	$\Delta s=0$
$L:-1$	0	0	-1	$\Delta L=0$
$B:0$	$+1$	$+1$	0	$\Delta B=0$

So, it is allowed reaction.

π^{-}	$\longrightarrow e^{-}$	$+\ \pi^{0}$	$+\ \nu_{e}$	
$Q:-1$	-1	0	0	$\Delta Q=0$
$spin:0$	$½$	0	$½$	$\Delta s\neq 0$

It is forbidden reaction.

μ^{-}	$+\ p$	$\longrightarrow n$	$+\ K^{+}$	$+\ K^{-1}$	
$Q:-1$	$+1$	0	$+1$	-1	$\Delta Q=0$
$spin:0$	$½$	$½$	0	0	$\Delta s=1$
$L:0$	0	0	0	0	$\Delta L=0$
$B:0$	1	1	0	0	$\Delta B=0$

It is allowed reaction.

π^{-}	$\longrightarrow e^{-}$	$+\ \bar{\nu}_{e}$	$+\ \nu_{\mu}$	
$Q:-1$	-1	0	0	$\Delta Q=0$
$spin:½$	$½$	$-½$	$½$	$\Delta s=0$
$L:1$	1	-1	1	$\Delta L=0$
$B:0$	0	0	0	$\Delta B=0$

It is allowed reaction

Correct option is (b)

19. Match the reactions on the left with the associated interactions on the right **[GATE 2010]**

(1) $\pi^+ \to \mu^+ + \nu_\mu$ (i) strong

(2) $\pi^0 \to \gamma + \gamma$ (ii) electromagnetic

(3) $\pi^0 + n \to \pi^- + p$ (iii) weak

(a) 1-iii, 2-ii, 3-i (b) 1-i, 2-ii, 3-iii (c) 1-ii, 2-i, 3-iii (d) 1-iii, 2-i, 3-ii

Soln. (i) Lepton are produced by weak interaction

(ii) γ-ray produced through electromagnetic interaction

(iii) Baryon are produced by strong interaction.

Correct option is (a)

20. The reaction ${}_1^2D + {}_1^2D \to {}_2^4He + \pi^0$ cannot proceed via strong interactions because it violates the conservations of **[NET June 2015]**

(a) angular momentum (b) electric charge (c) baryon number (d) isospin

Soln. ${}_1^2D + {}_1^2D \to {}_2^4He + \pi^0$

For strong interaction angular momentum, electric charge and Baryon number is conserved.

Therefore, Isospin is not conserved.

Correct option is (d)

21. If in the following reaction, the incident kaon has a kinetic energy of 1.63 GeV, calculate the total energy to be divided between the four recoiling particles.

$$\bar{K}^- + p^+ \to \Sigma^- + \pi^+ + \pi^- + \pi^+$$

The mass-energy of $\pi-$mesons are 139.6 MeV, $\Sigma^- = 1197.3$ MeV, proton = 938.3 MeV and $\bar{K}^- = 493.8$ MeV

Soln. Energy of particles taken together are:

$$\bar{K}^- = 0.4938 \text{ GeV} \quad (\because 1 \text{ GeV} = 10^3 \text{ MeV})$$

$$p^+ = 0.9383 \text{ GeV}$$

$$E_{\bar{K}^-} = 1.63 \text{ GeV}$$

$$\Rightarrow \quad \text{Total, } E = 3.0621 \text{ GeV}$$

Energy of the four recoiling particles are:

$$\Sigma^- = 1.1973 \text{ GeV}$$

$$\pi^+ = 0.1396 \text{ GeV}$$

$$\pi^- = 0.1396 \text{ GeV}$$

$$\pi^+ = 0.1396 \text{ GeV}$$

$$\text{Total energy} = 1.6161 \text{ GeV}$$

Therefore, excess energy = 3.0621 – 1.6161 = 1.446 GeV

Therefore, average energy per particle = 1.446/4 = 0.3615 GeV = 361.5 MeV

22. If a pion decays from rest to give a muon of 4.0 MeV energy, what is the kinetic energy of the accompanying neutrino? What is the mass of the neutrino in the process?

Soln. The decay mode of pion is given by, $\pi^- \to \mu^+ + v_\mu + E$

Therefore, energy, $E = (m_\pi - m_\mu) \times c^2$ (neutrino has zero 'rest' mass)

$$= (273m_e - 207m_e) \times c^2 = 66m_e \times c^2 = 66 \times 0.51 \text{ MeV}$$
$$= 33.7 \text{ MeV}$$

Since meon takes 4.0 MeV of energy, the kinetic energy of the accompanying neutrino is

$$(33.7 - 4.0) \text{MeV} = 29.7 \text{ MeV}$$

Therefore, mass of neturino = $(29.7/0.51)m_e = 58.23\ m_e$.

23. Find the value of third component of isotopic spin of Ξ^- in the following strong interaction:

$$\pi^+ + n \to \Xi^- + K^+ + K^+$$

Soln. The interaction is

$$\pi^+ + n \to \Xi^- + K^+ + K^+$$

$$I : 1 + \frac{1}{2} \to \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$I_3 : +1 - \frac{1}{2} \to I_3 + \frac{1}{2} + \frac{1}{2}$$

Therefore, I_3 for $\Xi^- = -\frac{1}{2}$

24. Identify the type of the following interaction from conservation laws:

$$\Sigma^0 \to \Lambda^0 + \gamma \quad (\text{life time} \leq 10^{-14}\text{ s})$$

Soln. We have $\Sigma^0 \to \Lambda^0 + \gamma$

Charge, Q	:	$0 \to 0 + 0$	$\Rightarrow \Delta Q = 0$
Baryon number, B	:	$+1 \to +1 + 0$	$\Rightarrow \Delta B = 0$
Lepton number, L	:	$0 \to 0 + 0$	$\Rightarrow \Delta L = 0$
Strangeness no. s	:	$-1 \to -1 + 0$	$\Rightarrow \Delta S = 0$
Hypercharge, Y	:	$0 \to 0 + 0$	$\Rightarrow Y = 0$

Since the strangeness number is conserved, the interaction is either a strong interaction or an electromagnetic one. Its half-life is $\leq 10^{-14}$s which points to the fact that it cannot be a strong interaction, but is a weak decay. As S is conserved, it cannot be a weak interaction. So, it is an electromagnetic interaction and a γ – photon is produced.

25. Identify the unknown particle in the reactions given below, using the conservation laws.

(i) $\mu^- + p \to {}_0^1n + ...$ (ii) $\pi^- + p \to K^0 + ...$

Soln. (i) The given reaction is : $\pi^- + p \to {}_0^1n + ...$

The unknown particle must be zero charge and mass, spin $\frac{1}{2}$ and lepton number 1 as they are conserved.

Since the interacting particle is π^- meson, the unknown particle is identified as mu-neutrino, v_μ

(ii) The reaction is $\pi^- + p \rightarrow K^0 + ...$

For charge conservation, Q : $-1+1 \rightarrow 0+Q$ $\Rightarrow Q=0$

Conservation of baryon no. B : $0+1 \rightarrow 0+Q$ $\Rightarrow B=+1$

Strangeness conservation, S : $0+0 \rightarrow -1+S$ $\Rightarrow S=+1$

Third component of isospin, I_3: $-1+\frac{1}{2} \rightarrow -\frac{1}{2}+I_3$ $\Rightarrow I_3=0$

Therefore, the unknown particle has charge zero, baryon number +1, strangeness number +1 and third component of isospin 0. So the particle could be Λ^0 or Σ^0.

26. Check if the following reactions are allowed or forbidden.

(i) $\pi^- + p \rightarrow \Lambda^0 + \pi^0$ (ii) $p + p^- \rightarrow 2\pi^+ + 2\pi^- + 2\pi^0$

Soln. (i) The reaction is : $\pi^- + p \rightarrow \Lambda^0 + \pi^0$

Q: $-1+1 \rightarrow 0+0$ $\Rightarrow \Delta Q = 0$ Q $\Rightarrow$ conserved

B: $0+1 \rightarrow +1+0$ $\Rightarrow \Delta B = 0$ B $\rightarrow$ conserved

S: $0+1 \rightarrow -1+0$ $\Rightarrow \Delta S \neq 0$ S $\Rightarrow$ not conserved

It is a strong interaction where the charge and baryon number are conserved. But since the strangeness number is not conserved, the reaction is forbidden.

(ii) The reaction is : $p + p^- \rightarrow 2\pi^+ + 2\pi^- + 2\pi^0$

Q: $+1-1 \rightarrow 2-2+0$; $\Delta Q = 0$

B: $+1-1 \rightarrow 0+0+0$; $\Delta B = 0$

S: $0+0 \rightarrow 0+0+0$; $\Delta S = 0$

Y: $+1-1 \rightarrow 0+0+0$; $\Delta Y = 0$

So, the above reaction is an allowed reaction.

27. An ultra-relativistic proton moves in a magnetic field. Can it radiate π^+, π^- and π^0, electrons and positrons?

Soln. If the energy of the proton is sufficiently large, it can radiate π^0 and π^+ mesons, and also positrons. The reactions are:

$$p \rightarrow p + \pi^0; \quad p \rightarrow n + \pi^+; \quad p \rightarrow n + e^+ \nu$$

But π^- mesons and electrons cannot be radiated.

28. Allocate the Isospin to the strange particles from following spins.

(a) $\pi^- + p \rightarrow \pi^0 + K^0$ (b) $p + p \rightarrow \pi^0 + K^+ + p$

(c) $\pi^+ + n \rightarrow \pi^0 + K^+$ (d) $\pi^- + p \rightarrow \Sigma^- + K^+$

(e) $\pi^+ + p \rightarrow \Sigma^+ + K^+$ (f) $\pi^+ + n \rightarrow \Xi^- + K^+ + K^+$

Soln. (a)

	π^-	+ p →	π^0	+ K^0
I_3	-1	½	0	-½

(b)

	p +	p →	π^0	+ K^+	+ p
I_3	½	½	0	½	½

(c)

	π^+ +	n →	π^0 +	K^+
I_3	+1	-½	0	½

(d)

	π^- +	p →	Σ^- +	K^+
I_3	-1	½	-1	½

(e)

	π^+	+ p →	Σ^+	+ K^+
I_3	1	½	+1	½

(f)

	π^+	+ n →	Ξ^-	+ K^+	+ K^+
I_3	1	$-½$	$-½$	½	½

29. Which of the following reactions are allowed and forbidden under the conservation of strangeness, conservation of baryon number and conservation of charge.

$\pi^+ + n \rightarrow \Lambda^0 + K^+$
$\pi^+ + n \rightarrow K^0 + K^+$
$\pi^+ + n \rightarrow \overline{K}^0 + \Sigma^+$
$\pi^+ + n \rightarrow \pi^- + p$
$\pi^- + p \rightarrow \Lambda^0 + K^0$
$\pi^- + p \rightarrow \pi^0 + \Lambda^0$

Soln. (i)

	π^+	+ n →	Λ^0	+ K^+	
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	1	0	$\Delta B = 0$
S	0	0	−1	1	$\Delta S = 0$

Allowed

(ii)

	π^+	+ n →	K^0	+ K^+	
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	0	0	$\Delta B \neq 0$
S	0	0	1	1	$\Delta S \neq 0$

Not allowed

(iii)

	π^+	+ n →	$\overline{K}^0$	+ Σ^+	
Q	1	0	0	1	$\Delta Q = 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	−1	−1	$\Delta S = 2$

Not allowed

(iv)

	π^+	+ n →	π^-	+ p	
Q	1	0	−1	1	$\Delta Q \neq 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	0	0	$\Delta S = 0$

Not allowed.

(v)

	π^-	+ p →	π^0	+ Λ^0	
Q	−1	1	0	0	$\Delta Q = 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	−1	1	$\Delta S = 0$

Allowed.

(vi)

	π^-	+ p →	π^0	+ Λ^0	
Q	−1	1	0	0	$\Delta Q = 0$
B	0	1	0	1	$\Delta B = 0$
S	0	0	0	−1	$\Delta S \neq 0$

Not allowed.

30. Calculate the energy of the neutron produced when a slow negative pion is captured by a proton. Should neutron be treated relativistically?

Soln. $\pi^- + p^+ \rightarrow n^0 + \gamma + Q$

$139 + 938 \rightarrow 939 + h\nu + Q$ or $Q = 138\,MeV = E_\gamma + E_n$

From conservation of momentum $m_n v_n = E_\gamma / c$

and $\dfrac{E_n}{E_\gamma} = \dfrac{1}{2}\dfrac{m_n v_n^2}{c m_n v_n} = \dfrac{1}{2}\dfrac{v_n}{c} = \dfrac{1}{2}\dfrac{E_\gamma}{m_n c^2} = \dfrac{E\gamma}{1878}$

or $\frac{E_n}{E_\gamma + E_n} = \frac{E_\gamma}{1878 + E_\gamma} \therefore \frac{E_n}{138} = \frac{E_\gamma}{1878 + E_\gamma}$

and $\frac{E_n}{138} = \frac{138 - E_n}{1878 + 138 - E_n} = \frac{138 - E_n}{2016 - E_n}$

or $E_n = 9MeV$

Using relativistic relations

$m_n v_n = m_{0n} v_n / (1 - \beta^2)^{1/2} = E_\gamma / c$ and $E_n = m_{0n} c^2 [(1 - \beta^2)^{-1/2} - 1]$

We get $E_n = 8.8MeV$

31. Classify the following processes in terms of the type of interaction.

Soln: $\pi^- + p \to \Lambda^0 + K^0$; $\pi^- + p \to \pi^0 + n$; $p + \gamma \to p + \pi^0$; $\Sigma^0 \to \Lambda^0 + \gamma$; $\pi^0 \to \gamma + \gamma$; $K^0 \to \pi^+ + \pi^-$; $\Lambda^0 \to p + \pi^-$; $\Xi^- \to \Lambda^0 + \pi^-$; and $\Lambda^0 \to p + e^- + \bar{\nu}$.

In the reactions $\pi^- + p \to \Lambda^0 + K^0$ and $\pi^- + p \to \pi^0 + n$, the interaction is a short-range force between Hadrons (π, p, n, Λ and K) corresponding to one pion-exchange. These reactions obey selection rules $\Delta B = 0, \Delta Q = 0, \Delta Y = 0, \Delta\pi = 0, \Delta T = 0, \Delta T_3 = 0$

hence the interaction is strong.

In the reactions $p + \gamma \to \pi^0 + p$, $\Sigma^0 \to \Lambda^0 + \gamma$ and $\pi^0 \to \gamma + \gamma$, the interaction is electromagnetic, as the electromagnetic force arises due to the mechanical effect of the emission and absorption of virtual photons. These reactions obey all above mentioned selection rules except the rule for conservation of isospin T, hence, the interaction in these processes is electromagnetic.

In the reactions $K^0 \to \pi^+ + \pi^-$, $\Lambda^0 \to p + \pi^-$, $\Xi^- \to \Lambda^0 + \pi^-$ and $\Lambda^0 \to p + e^- + \bar{\nu}$, the interaction is weak, as the weak interaction is due to the leptonic decay of strange and non-strange particles and due to non-leptonic decay of strange particles. These reactions do not conserve parity, strangeness, isospin and the third component of isospin, hence the interaction in these processes is weak.

32. What are the possible values of isotopic spin for the following systems? (a) A π^+ meson and an antiproton, (b) two neutrons, (c) a π^+ meson and a Λ^c, (d) a π^+ and a π^0 meson, (e) a u and a $\bar{u}$ quark, (f) a c, b and an s quark.

Soln. $I(u) = \frac{1}{2} = I(d)$ while $I(s) = I(c) = I(b) = I(t) = 0$

(a) Using the composition law for isospin, we get

$I(\pi^+\bar{p}) = \frac{3}{2}, \frac{1}{2}$, $I_3(\pi^+\bar{p}) = I_3(\pi^+) + I_3(\bar{p}) = 1 - \frac{1}{2} = \frac{1}{2}$

Since $I_3 = \frac{1}{2}$ can be a legitimate isospin projection for both $I = \frac{3}{2}$ and $I = \frac{1}{2}$, we conclude that the composite state $(\pi^+\bar{p})$ can have either $I = \frac{3}{2}$ or $I = \frac{1}{2}$.

(b) A composite state of two neutrons can have $I(nn) = 1, 0$

$I_3(nn) = I_3(n) + I_3(n) = -\frac{1}{2} - \frac{1}{2} = -1.$

Since $I_3 = -1$ cannot be an isospin projection of a state with $I = 0$, we conclude that a composite state of two neutrons can only be $I = 1$.

(c) The Λ^0 particle is an isosinglet ($I = 0$) and, consequently, we have

$I(\pi^+\Lambda^0) = 1$

This composite state is therefore $I = 1$.

(d) We can have $I(\pi^+\pi^0) = 2,1,0$

$I_3(\pi^+\pi^0) = I_3(\pi^+) + I_3(\pi^0) = 1 + 0 = 1$

A state with two π mesons can have $I = 2$, $I = 1$ or $I = 0$. However, $I = 0$ has no projection of $I_3 = 1$, and the composite state $(\pi^+\pi^0)$ must therefore have $I = 2$ or $I = 1$.

(e) For a composite system of a u and a $\bar{u}$ quark, we have

$I(u\bar{u}) = 1,0$

$$I_3(u\bar{u}) = I_3(u) + I_3(\bar{u}) = \frac{1}{2} - \frac{1}{2} = 0$$

Since the projection $I_3 = 0$ is possible for both $I = 1$, 0 states, we conclude that the composite state $(u\bar{u})$ can have $I = 1$ or $I = 0$.

(f) All quarks with flavor quantum numbers are isosinglets. Thus, we have

$I(bcs) = 0$

This composite state would therefore be a baryon with unique isospin $I = 0$.

PRACTICE SET

1. Consider the four processes

(i) $p^+ \to n + e^+ + \nu_e$ (ii) $\Lambda^0 \to p^+ + e^+ + \nu_e$

(iii) $\pi^+ \to e^+ + \nu_e$ (iv) $\pi^0 \to \gamma + \gamma$

Which of the above is/are forbidden for free particles?

(a) only (ii) (b) (ii) and (iv) (c) (i) and (iv) (d) (i) and (ii)

2. Consider the following ratios of the partial decay widths $R_1 = \frac{\Gamma(\rho^+ \to \pi^+ + \pi^0)}{\Gamma(\rho^- \to \pi^- + \pi^0)}$ and $R_2 = \frac{\Gamma(\Delta^{++} \to \pi^+ + p)}{\Gamma(\Delta^- \to \pi^- + n)}$. If the effects of electromagnetic and weak interactions are neglected, then R_1 and R_2 are, respectively,

(a) 1 and $\sqrt{2}$ (b) 1 and 2 (c) 2 and 1 (d) 1 and 1

3. Consider the following particles: the proton p, the neutron n, the neutral pion π^0 and the delta resonance Δ^+. When ordered in terms of decreasing lifetime, the correct arrangement is as follows:

(a) π^0, n, p, Δ^+ (b) p, n, Δ^+, π^0 (c) p, n, π^0, Δ^+ (d) Δ^+, n, π^0, p

4. A pion π^+ of rest mass 140 MeV/c^2 is accelerated to an energy of 2.5 GeV. This cannot decay to a proton of rest mass 938 MeV/c^2 and an anti-neutron of rest mass 939 MeV/c^2 because

(a) a moving pion cannot produce a proton at rest
(b) energy and momentum cannot be simultaneously conserved
(c) the spin of the pion is different from the spin of the proton
(d) it is forbidden by isospin conservation

5. Which of the following quantities of an elementary particle should be zero if parity is conserved?

(a) mass width (b) magnetic dipole moment
(c) electric quadrupole moment (d) electric dipole moment

6. The prinicipal decay mode of a neutral pion π^0 is through the process

(a) $\pi^0 \to \gamma\gamma$ (b) $\pi^0 \to e^+e^-$ (c) $\pi^0 \to \mu^+\mu^-$ (d) $\pi^0 \to \nu\bar{\nu}$

7. Which one of the following sets corresponds to fundamental particles ?

(a) proton, electron and neutron (b) proton, electron and photon
(c) electron, photon and neutrino (d) quark, electron and meson

8. A delta baryon is found to have charge +2 and strangeness 0. Its isospin must be

(a) 2 (b) 3/2 (c) 1 (d) $\frac{1}{2}$

9. A muon participates is

(a) electromagnetic, weak and gravitational interactions only
(b) weak and gravitational interactions only
(c) weak and strong interaction only
(d) electromagnetic and gravitational interactions only